Do Mirrors for Gravitational Waves Exist?

Stephen Minter[a], Kirk Wegter-McNelly[b], and Raymond Chiao[c]

[a]University of California, Merced, School of Natural Sciences, P.O. Box 2039, Merced, CA 95344, USA

[b]Boston University, School of Theology, 745 Commonwealth Ave., Boston, MA 02215, USA

[c]University of California, Merced, Schools of Natural Sciences and Engineering, P.O. Box 2039, Merced, CA 95344, USA.
Corresponding author, email: rchiao@ucmerced.edu

Abstract

Thin superconducting films are predicted to be highly reflective mirrors for gravitational waves at microwave frequencies. The quantum mechanical *non-localizability* of the negatively charged Cooper pairs, which is protected from the localizing effect of decoherence by an energy gap, causes the pairs to undergo *non-picturable*, *non-geodesic* motion in the presence of a gravitational wave. This non-geodesic motion, which is accelerated motion *through* space, leads to the existence of mass and charge supercurrents inside the superconducting film. On the other hand, the decoherence-induced *localizability* of the positively charged ions in the lattice causes them to undergo *picturable*, *geodesic* motion as they are carried along *with* space in the presence of the same gravitational wave. The resulting separation of charges leads to a virtual plasma excitation within the film that enormously enhances its interaction with the wave, relative to that of a neutral superfluid or any normal matter. The existence of strong mass supercurrents within a superconducting film in the presence of a gravitational wave, dubbed the "Heisenberg-Coulomb effect," implies the specular reflection of a gravitational microwave from a film whose thickness is much less than the London penetration depth of the material, in close analogy with the electromagnetic case. The argument is developed by allowing classical gravitational fields, which obey Maxwell-like equations, to interact with quantum matter, which is described using the BCS and Ginzburg-Landau theories of superconductivity, as well as a collisionless plasma model. Several possible experimental tests of these ideas, including mesoscopic ones, are presented alongside comments on the broader theoretical implications of the central hypothesis.

Keywords: gravitational wave, mirror, superconductor, uncertainty principle, equivalence principle, Heisenberg-Coulomb effect

PACS: 04.30.Nk, 04.80.Nn, 74.78.-w, 52.30.-q, 84.40.-x

I. INTRODUCTION

Experiments at the frontiers of quantum mechanics and gravitation are rare. In this paper we argue for a claim that may lead to several new types of experiment, namely, that a superconducting film whose thickness is less than the London penetration depth of the material can specularly reflect not only electromagnetic (EM) microwaves, as has been experimentally demonstrated [1, 2], but gravitational (GR) microwaves as well. The basic motivation for our approach lies in the well-known fact that Einstein's field equations lead, in the limits of *weak* GR fields and *non-relativistic* matter, to gravitational Maxwell-like equations [3], which in turn lead to boundary conditions for gravitational fields at the surfaces of the superconducting films homologous to those of electromagnetism. All radiation fields, whether electromagnetic or gravitational, will be treated classically, whereas the superconductors with which they interact will be treated quantum mechanically. Thus, in this paper we adopt a *semi-classical* approach to the interaction of gravitational radiation with matter.

Not enough effort has been made to investigate the ramifications of the gravitational Maxwell-like equations for the interaction of GR waves with matter, perhaps because the so-called "electromagnetic analogy" has been so hotly contested over the years [4]. In any case, we believe that these equations provide a helpful framework for thinking about the response of non-relativistic matter to weak, time-varying gravitational fields, especially that of macroscopically coherent quantum charge and mass carriers, namely, the Cooper pairs of conventional, type I superconductors. We argue here that the electromagnetic analogy manifested in the Maxwell-like equations implies that type I superconductors can be surprisingly efficient mirrors for GR waves at microwave frequencies.

In Section 2, we introduce the two basic claims upon which the larger argument rests. Together, these two claims open the door to an enormously enhanced interaction between a GR microwave and a type I superconductor, relative to what one would expect in the case of a neutral superfluid or, indeed, any normal metal or other classical matter. The first claim is that a GR microwave will generate quantum probability supercurrents, and thus mass and electrical supercurrents, inside a type I superconductor, due to the quantum mechanical *non-localizability* of the Cooper pairs within the material.

The non-localizability of Cooper pairs, which is ultimately due to the Uncertainty Principle (UP), causes them to undergo *non-picturable*, *non-geodesic* motion in the presence of a GR wave. This non-geodesic motion, which is accelerated motion *through* space, leads to the existence of mass and charge supercurrents inside a superconductor. By contrast, the localizability of the ions within the superconductor's lattice causes them to undergo *picturable*, *geodesic* motion, i.e., free fall, in

the presence of the same wave. The resulting relative motion between the Cooper pairs and the ionic lattice causes the electrical polarization of the superconductor in the presence of a GR wave, since its Cooper pairs and ions carry not only mass but oppositely signed charge as well.

Furthermore, the non-localizability of the Cooper pairs is "protected" from the normal process of localization, i.e., from decoherence, by the characteristic energy gap of the Bardeen-Cooper-Schrieffer (BCS) theory of superconductivity. The decoherence of entangled quantum systems such as Cooper pairs (which are in the spin-singlet state) is the fundamental cause of the *localizability* of all normal matter [5]. Indeed, this "classicalizing" process must occur within any spatially extended system before the idea of the "universality of free fall" [6] can be meaningfully applied to its parts. After all, the classical principle behind the universality of free fall, the Equivalence Principle (EP), is a strictly *local* principle [7].

The second of the two claims presented in Section 2 is that the mass supercurrents induced by a GR wave are much stronger than what one would expect in the case of a neutral superfluid or any normal matter, due to the electrical polarization of the superconductor caused by the wave. This is what we refer to as the "Heisenberg-Coulomb (H-C) effect." The magnitude of the enhancement due to the H-C effect (derived in Section 7) is given by the ratio of the electrical force to the gravitational force between two electrons,

$$\frac{e^2}{4\pi\varepsilon_0 G m_e^2} = 4.2 \times 10^{42} \ , \tag{1}$$

where e is the electron charge, m_e is the electron mass, ε_0 is the permittivity of free space, and G is Newton's constant. The enormity of (1) implies the possibility of an enormous back-action of a superconductor upon an incident GR wave, leading to its reflection.

Of the four fundamental forces of nature, viz., the gravitational, the electromagnetic, the weak, and the strong forces, only gravity and electricity have long range, inverse square laws. The pure number obtained in (1) by taking the ratio of these two inverse-square laws is therefore just as fundamental as the fine structure constant. Because this number is so large, the gravitational force is typically ignored in treatments of the relevant quantum physics. But as we shall see below, a semi-classical treatment of the interaction of a superconductor with a GR wave must account for both the electrodynamics and the gravito-electrodynamics of the superconductor, since both play an important role in its overall response to a GR wave.

In Section 3, we consider the interaction between an EM wave and a thin metallic film having an arbitrary, frequency-dependent complex conductivity. We determine the relevant boundary conditions using Faraday's and Ampere's laws in order to derive general expressions for the transmissivity and reflectivity of a thin film. In Section 4, we show that, in the case of a superconducting film, the

BCS theory implies that EM waves at microwave frequencies will be specularly reflected even from films whose thickness is less than the London penetration depth of the material, or, equivalently (at sufficiently low frequencies), less than the material's plasma skin depth, as has been experimentally observed [1, 2]. We show, furthermore, that the frequency at which reflectivity drops to 50%, what we call the "roll-off frequency" ω_{r}, depends only on the ratio of the speed of light c to a single parameter, the length scale l_{k} associated with the kinetic inductance L_{k} of the film's Cooper pairs [8], which in turn depends on the plasma skin depth δ_{p}. In the electromagnetic case, the microscopic size of δ_{p} leads to a microscopic value for l_{k} and thus to the possibility of specular reflection over a wide range of frequencies (including microwave frequencies) in the EM case.

In Section 5, we review the Maxwell-like equations for linearized Einsteinian gravity and highlight the fact that any normal matter, with its inherently high levels of dissipation, will necessarily be an inefficient reflector of GR waves because of its high impedance relative to the extremely low "gravitational characteristic impedance of free space" Z_{G} (2.8×10^{-18} in SI units). Superconductors, on the other hand, are effectively *dissipationless* at temperatures near absolute zero because of their quantum mechanical nature [2]. The fact that a superconductor's effectively *zero* impedance can be much smaller than the very small quantity Z_{G} allows it to reflect an incoming GR wave, much as a low-impedance connection or "short" at the end of a transmission line can reflect an incoming EM wave.

In Section 6, we appeal to the Maxwell-like equations introduced in Section 5, to the identicality of the boundary conditions that follow from them, and to the linearity of weak GR-wave optics, in order to introduce GR analogs of the earlier EM expressions for the reflectivity and roll-off frequency. As in the EM case, the GR roll-off frequency $\omega_{\mathrm{r,G}}$ can be expressed as the ratio of the speed of light c to a single parameter. In this case, however, the relevant parameter is the length scale $l_{\mathrm{k,G}}$ associated with the *gravitational* kinetic inductance $L_{\mathrm{k,G}}$ of the Cooper pairs. In this section we treat the superconductor as if it were a neutral superfluid, i.e., as if its Cooper pairs were electrically neutral particles interacting with one another and the with ionic lattice exclusively through their mass. Although this assumption is unphysical, it leads to a result in agreement with conventional wisdom, namely, that the gravitational plasma skin depth $\delta_{\mathrm{p,G}}$ and the kinetic inductance length scale $l_{\mathrm{k,G}}$ will be astronomical in size ($\sim 10^{13}$ m and $\sim 10^{36}$ m, respectively). Such enormous values imply that $\omega_{\mathrm{r,G}}$ will be effectively zero, and thus that superconductors cannot function as mirrors for GR microwaves in laboratory-scale experiments.

In Section 7, we show why the approach taken at the end of the previous section, in accord with conventional wisdom, is wrong. Superconductors *can* function as laboratory-scale mirrors for GR microwaves because of the H-C effect. When one takes into account the electrical charge separation

induced within a superconductor by a GR wave (due to the BCS-gap-protected non-localizability of its Cooper pairs), the ratio given in (1) enters into the analysis in such a way as to keep $l_{\mathrm{k,G}}$ microscopic and to raise $\omega_{\mathrm{r,G}}$ to the level of ω_{r}. Thus the H-C effect greatly enhances the reflection of a GR wave from the surface of a superconductor – by 42 orders of magnitude! – relative to what one would expect from a neutral superfluid, a normal metal, or any normal matter.

Because both charge supercurrents and mass supercurrents are generated by an incoming GR wave (and by an incoming EM wave), it is also necessary to consider whether superconducting films are not mirrors but rather transducers, i.e., converters, of GR radiation into EM radiation (in the case of an incident GR wave), or vice versa (in the case of an incident EM wave). In Section 8, we take up this particular question and show that transduction in both directions is too weak to decrease reflection by any appreciable amount. In section 9, however, we show that energy is conserved only when transduction is included in the overall analysis as an effective absorption mechanism.

Finally, in Section 10 we indicate several possible experimental tests of the basic claims advanced in the paper and offer brief comments on the broader theoretical implications of our central hypothesis. Whereas present GR-wave experiments aim to passively detect GR waves originating from astrophysical sources, our argument implies the possibility of several new types of laboratory-scale experiment involving GR waves. One type would test the physics behind the Heisenberg-Coulomb effect by looking for a departure from geodesic motion in the case of two coherently connected superconducting bodies that are allowed to fall freely through a distance large enough to observe tidal effects. A second type would investigate the existence and strength of any gravitational Casimir-like effect between two type I superconductors. Yet a third type, involving an electrically charged pair of superconductors, would allow for more direct investigation of the existence and properties of GR-waves, the results of which would bear significantly on the search for a quantized theory of gravity.

Three appendices address ancillary issues: (A) the relationship between the magnetic and kinetic inductances of a thin film, (B) the kinetic inductance length scale according to a collisionless plasma model, and (C) the relationship between the impedance argument given in Section 5 and Weinberg's argument regarding the scattering cross-section of a Weber-style resonant bar antenna, including an application of the Kramers-Kronig relations to the sum rule for the strength of the interaction between a GR wave and a superconductor.

II. THE UNCERTAINTY PRINCIPLE LIMITS THE APPLICABILITY OF THE EQUIVALENCE PRINCIPLE

It is helpful to begin the analysis with a simple model of the interaction between a weak GR wave and a normal metallic film. For the sake of eventually considering the possibility of mirrors (i.e., the possibility of "ray" optics), we will assume here and throughout that the lateral dimensions of the film are very large when compared to the wavelength of the incident wave. Focusing on waves with very high frequencies, i.e., microwaves, will allow us to treat the ions and normal electrons of a laboratory-scale film as though they were freely floating, non-interacting "dust" or point particles undergoing free fall along classical trajectories, i.e., traveling along geodesics.

Although it would be possible in principle, in this approximation, to detect the passage of a GR wave over the film by observing the geodesic deviation among its different components (the principle underlying LIGO), the film *cannot*, in this approximation, interact energetically with a very high frequency GR wave. It cannot absorb or scatter any of the wave's energy because each of its localized particles must, according to the EP, travel along a geodesic, i.e., each particle must remain at rest with respect to its local, co-moving, and freely-falling inertial frame [9]. And since there can be no energetic interaction with the wave, mass currents cannot be generated locally within the film without violating the conservation of energy.

It is true that a distant inertial observer will see the "dust" particles undergo quadrupolar motion, and will thus expect the film to emit GR radiation. But this apparent paradox can be resolved by noting that the wave causes the film's ions and normal electrons (which are to be treated as *test* particles whose masses and gravitational fields are negligible) to be carried along *with* space rather than accelerated *through* space. Only the latter kind of motion, in which the wave does work on the particles, and hence transfers kinetic energy to them, leads to the time-varying mass quadrupole moment that enters into Einstein's quadrupole formula for the emission of GR radiation (see Figure 1).

The classical concept of a "geodesic" depends fundamentally upon the localizability, or spatial separability, of particles. From a quantum mechanical point of view, localizability arises ultimately from the *decoherence of entangled states*, i.e., from the "collapse" of nonfactorizable superpositions of product wavefunctions of two or more particles located at two or more spatially well-separated points in space, into spatially separable, factorizable, product wavefunctions, upon the interaction of the particles with their environment. Decoherence typically occurs on extremely short time-scales due to the slightest interaction with the environment [5]. Whenever it does occur, one can speak classically of point particles having trajectories or traveling along geodesics. Only *after* decoherence

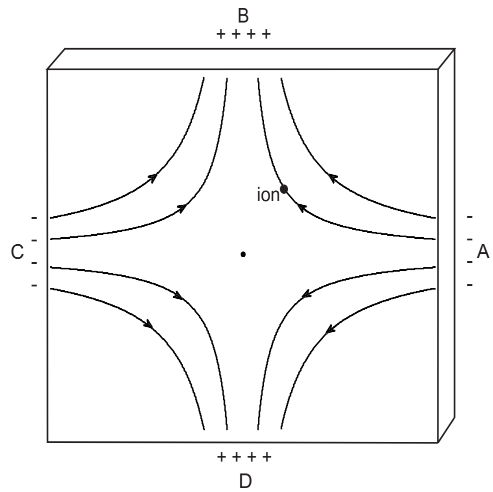

FIG. 1: A snapshot of a square metallic plate with a very high frequency GR wave incident upon it at the moment when the gravitational tidal "forces" on the plate are those indicated by the hyperbolae, as seen by a distant observer. All ions, being approximately in free fall, are carried along *with* space rather than accelerated *through* space. No work is done on them, and thus no kinetic energy is transferred to them, by the wave. When the metal in the plate is normal, all ions and all normal electrons locally co-move together along the same geodesics in approximate free fall, so that the plate remains neutral and electrically unpolarized. However, when the plate becomes superconducting, the Cooper pairs, being in non-local *entangled* states, remain at rest with respect to the center of mass according to the distant observer, and do not undergo free fall along with the ions and any residual normal electrons. This non-picturable, non-geodesic, *accelerated* motion of the Cooper pairs *through* space leads to *picturable* quantum probability supercurrents, which follow the same hyperbolae as the incident tidal GR wave fields (see Eqs. (62)-(69)). Since the Cooper pairs carry not only mass but also charge, both mass and electrical supercurrents are generated, and both types of current carry energy extracted from the gravitational wave. In the snapshot shown, this leads to the accumulation of positive charge at B and D, and to the accumulation of negative charge at A and C, i.e., to a quadrupolar-patterned electrical polarization of the superconductor. The resulting enormous Coulomb forces strongly oppose the effect of the incoming tidal gravitational fields, resulting in the mirror-like reflection of the incoming GR wave.

has occurred does the Equivalence Principle become a well-defined principle, for only then does a particle's geodesic become well defined. In other words, only through decoherence does the law of the "universality of free fall," i.e., the experimentally well-established claim that "the gravitational acceleration of a point body is independent of its composition" [6], become meaningful.

Entangled quantum states imply the nonlocalizability of particles, in the sense that such states lead to experimentally well-confirmed violations of Bell's inequalities [10, Chapters 6 and 19]. We claim here that Cooper pairs are completely non-localizable within a superconductor, not only in the sense of Heisenberg's Uncertainty Principle, but also because each electron in a given Cooper

pair in the BCS ground state is in an entangled state, since each pair is in a superposition state of the product of two electron wavefunctions with opposite momenta, and also simultaneously in a superposition state of the product of two opposite electron spin-1/2 states (i.e., a spin-singlet state). The violation of Bell's inequalities by these entangled states in the BCS ground state means that this state is *non-local*, in the sense that instantaneous correlations-at-a-distance between the two electrons of a given Cooper pair must occur in the superconductor upon remote measurements within a long, single continuous piece of superconductor (the distance between these remote measurements can be arbitrarily large). Although these instantaneous correlations-at-a-distance cannot be used to send signals faster than light [10], they also cannot be accounted for in any local, realistic theory of quantum phenomena, including those which satisfy the completeness conditions demanded by Einstein, Podolsky, and Rosen (EPR) [11].

The localizability or spatial separability of *all* particles, as envisioned by EPR, would of necessity lead to the *universal validity* of the Equivalence Principle, and thus to the idea that even Cooper pairs must undergo geodesic motion (i.e., free-fall) within a superconductor in response to an incident GR wave. There could be no relative motion between the Cooper pairs and the ions, no spatial separation of charges inside the superconductor, and no enhancement, even in principle, of the superconductor's interaction with a GR wave relative to that of a normal metal interacting with the same wave. But Cooper pairs are manifestly *not* localizable within the superconductor, since they are fully quantum mechanical, non-local systems. For this reason the "dust-particles-following-geodesics" model introduced earlier must fail in the case of a superconductor, even as a first approximation [12].

When a conventional, type I superconductor is in the BCS ground state, each of its Cooper pairs is in a zero-momentum eigenstate relative to the center of mass of the system. According to Heisenberg's Uncertainty Principle (UP), the fact that the Cooper pairs' momenta are perfectly definite entails that their positions within the superconductor are completely *uncertain*, i.e., that the pairs are *non-localizable*. The motion of a given Cooper pair within the superconductor is irreducibly quantum mechanical in nature, being related to the pair's wavefunction. Such motion *cannot* be pictured in terms of a well-defined trajectory or geodesic [13]. Indeed, at a conceptual level, the ascription of a "trajectory" or "geodesic" to a given Cooper pair within a superconductor becomes *meaningless* in the BCS ground state. This is similar to what Bohr taught us concerning the meaninglessness of the concept of "orbit" in the ground state of the hydrogen atom during its interaction with radiation fields [14, pp. 113ff].

The robustness of the BCS ground state in the face of perturbations is guaranteed by the BCS energy gap, which "protects" the Cooper pairs from making quantum transitions into excited states,

such as happens in pair-breaking (as long as the material is kept well below its transition temperature and the frequency of the incident radiation is below the BCS gap frequency [15]). The energy gap prevents the pairs from decohering, and from becoming localized like the superconductor's ions and any residual, normal conduction electrons [16]. If the Cooper pairs cannot be thought of as *localizable* point bodies, then the "universality" of free fall cannot be meaningfully applied to them. In short, an application of the EP to the motion of Cooper pairs within a superconductor is fundamentally *precluded* by the UP. This is not to make the well-known point that quantum field theories may lead to measurable "quantum violations of the EP" due to possible "fifth-force" effects that produce slight corrections to particle geodesics (see, for example, Adelberger [6] and Ahluwalia [17]), but rather to observe that the non-localizability of quantum objects places a fundamental limit on the *applicability* of the EP (a point previously raised by Chiao [18, esp. Section V]).

In contrast to a superconductor's non-localizable Cooper pairs, its ions (and, at finite temperatures, any residual background of normal electrons) are unaffected by the energy gap, and are thus fully *localized* by the decohering effect of their interactions with the environment. Thus, unlike Cooper pairs, the ionic lattice possesses no coherent quantum phase anywhere. The *geodesic* motion of the ions will therefore differ from the *non-geodesic* motion of the Cooper pairs. The latter, which is accelerated motion *through* space, implies the existence of quantum probability supercurrents, and thus of mass and electrical supercurrents, inside the superconductor (see Figure 1). These supercurrents will carry energy extracted from the GR wave. The possibility of a non-negligible energetic interaction between a GR wave and a superconductor depends crucially upon this initial claim, which is implied by the absence of the localizing effect of decoherence upon the Cooper pairs.

Before we turn to the second claim, it is worth noting that the non-geodesic motion of a superconductor's Cooper pairs also follows from what London called the "rigidity of the wavefunction" [19]. The phase of the wavefunction of each Cooper pair must be constant in the BCS ground state prior to the arrival of a GR wave. This implies that the gradient of its phase is initially zero. Since an incoming GR wave whose frequency is less than the BCS gap frequency cannot alter this phase (in the lowest order of time-dependent perturbation theory), and since the canonical momentum of any given pair relative to the center of mass of the superconductor is proportional to the gradient of its phase, the canonical momentum of each pair must remain zero at all times with respect to the center of mass of the system in the presence of a GR wave, as seen by the distant inertial observer.

This quantum-type rigidity implies that Cooper pairs will acquire kinetic energy from a GR wave in the form of a nonzero *kinetic* velocity, i.e., that they will be *accelerated* by the wave relative to any local inertial frame whose origin does not coincide with the center of mass of the system (for

example, at the corners of a large, square superconducting film; see Section 7). In other words, the apparent "motionlessness" of the Cooper pairs in the presence of a GR wave, as witnessed by a distant inertial observer, in fact entails their accelerated motion *through* local space. Again, this behavior implies the existence of mass supercurrents inside the superconductor that carry energy extracted from the wave.

Of course, even normal matter such as in a Weber-style resonant bar detector has some extremely small degree of rigidity arising from its very weak interatomic coupling. Thus normal matter does not, strictly speaking, behave as a collection of freely falling, noninteracting "dust particles" in the presence of a very low frequency GR wave. Instead, like the Cooper pairs, but to a much smaller degree, and at much lower frequencies than the microwave frequencies being considered here, normal matter opposes the squeezing and stretching of space going on around it (as Feynman pointed out in his well-known remarks on why GR waves must carry energy [20]). Thus, even normal matter will acquire an extremely small amount of kinetic energy as it is accelerated through space by a passing GR wave. In this case, though, high levels of dissipation inside the material will cause whatever small amount of energy is extracted from the GR wave to be overwhelmingly converted into heat instead of being predominantly re-radiated as a scattered GR wave (as Weinberg has pointed out [21]). A key feature of the mass supercurrents carried by Cooper pairs is that they are *dissipationless*. We shall return to this particular point in Section 5.

The second basic claim underlying the paper's larger argument follows from the dual nature of the supercurrents generated by a GR wave within a superconductor. Since a GR wave will generate both mass and charge supercurrents, it will *electrically polarize* the superconductor. This important observation implicates the Coulomb force of attraction between the oppositely signed charges that must accumulate at the edges of the superconductor, if there is to be no violation of charge conservation (see Figure 1). These oppositely signed charges will consist of negatively charged Cooper pairs, on the one hand, and corresponding, positively charged Cooper-pair holes (hereafter, "holes"), on the other. An incoming GR wave with a frequency well below the superconductor's plasma frequency will thus generate a virtual plasma excitation inside the superconductor. The resulting *Coulomb* force between the Cooper pairs and holes, which acts as a Hooke's law restoring force, strongly opposes the effect of the incident wave. The enormous back-action of this force on the motion of the Cooper pairs greatly enhances their *mass* conductivity (see Section 7), to the point where specular reflection of an incident GR wave from a superconducting film becomes possible. The existence of strengthened mass supercurrents within a superconductor, which is due to the combined effect of the quantum non-localizability of the Cooper pairs and the Coulomb attraction between the pairs and holes, is what we refer to as the "Heisenberg-Coulomb effect."

Consider, by way of contrast, what happens when a GR wave impinges on a superfluid, whose constituent particles are electrically *neutral.* Mass supercurrents will again be induced by the wave, due to quantum non-localizability, but in this case there will be no enhancement effect because the mass carriers within a superfluid are its electrically neutral atoms. Thus no appreciable fraction of incident GR-wave power can be reflected from the surface of a neutral superfluid. On the other hand, one might worry that the size of the H-C effect in a superconductor would drive its mass supercurrents above the critical level, thereby undermining the possibility of specular reflection. But it should always be possible to arbitrarily reduce the amplitude of the driving radiation field until the superconductor responds *linearly* to the field (see the related discussion of superluminality at the end of Section 7). The existence of a linear-response regime guarantees the possibility of fabricating *linear* GR-wave optical elements, including mirrors.

III. THE INTERACTION OF AN EM WAVE WITH A THIN METALLIC FILM

The question of the interaction of an EM wave with a metallic film whose thickness d is small compared to the wavelength can be addressed using "lumped-circuit" concepts such as resistance, reactance, inductance, etc., of an infinitesimal square element of the film. (As before, we assume, for the sake of considering mirror-like behavior, that the lateral dimensions of the film are at least on par with the wavelength of the incident wave.) In this section we derive a formula for the transmissivity $\mathcal{T}$ as well as the reflectivity $\mathcal{R}$ of a thin metallic film with an arbitrary, frequency-dependent complex conductivity. In the next section we apply this analysis to the case of a *superconducting* film.

The complex amplitude reflection coefficient r corresponding to the proportion of incident EM radiation at frequency ω reflected from a thin film and the complex amplitude transmission coefficient t corresponding to the proportion of the same radiation transmitted through the film can be defined as follows:

$$\mathbf{E}_{\text{reflected}} = r\mathbf{E}_{\text{incident}} \tag{2a}$$

$$\mathbf{E}_{\text{transmitted}} = t\mathbf{E}_{\text{incident}} \ . \tag{2b}$$

By convention, r, if it is real, is defined to be *positive* when the reflected electric field $\mathbf{E}_{\text{reflected}}$ is *oppositely* directed to the incident electric field $\mathbf{E}_{\text{incident}}$. On the other hand, t, if it is real, is defined to be *positive* when the transmitted electric field $\mathbf{E}_{\text{transmitted}}$ points in the *same* direction as the incident electric field $\mathbf{E}_{\text{incident}}$. In general, r and t are complex quantities whose values depend on the frequency ω of the incident wave, but all radiation fields will be treated classically.

Since the tangential components of the electric fields must be continuous across the vacuum-film

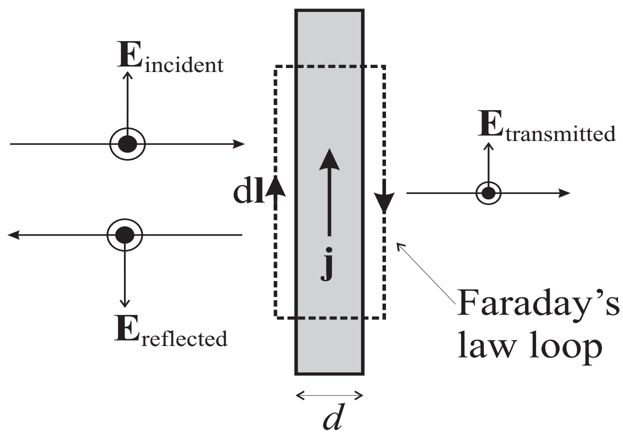

FIG. 2: A thin metallic film of thickness d is straddled by a rectangular loop (dashed lines) for applying Faraday's law to it. An incident EM wave is partially transmitted and partially reflected by the film. The EM wave generates an electrical current density $\mathbf{j}$, which flows uniformly inside the film. A similar rectangular loop (not shown) lying in a plane parallel to the magnetic fields (denoted by the circles with central dots) is for applying Ampere's law.

interface, the electric field inside the film $\mathbf{E}_{\text{inside}}$ drives a current density $\mathbf{j}$ inside the film that is *linearly* related to this driving electric field, for the general case of a *linear-response* theory of the interaction of matter with *weak* driving fields. This linear relationship is given by

$$\mathbf{j}(\omega) = \sigma(\omega)\mathbf{E}_{\text{inside}}(\omega), \text{ where} \tag{3}$$

$$\mathbf{E}_{\text{inside}} = (1-r)\mathbf{E}_{\text{incident}} \quad \text{at frequency } \omega. \tag{4}$$

In general, the conductivity $\sigma(\omega)$ associated with the current generated within the film at a given driving frequency ω will be a complex quantity:

$$\sigma(\omega) = \sigma_1(\omega) + i\sigma_2(\omega), \tag{5}$$

where $\sigma_1(\omega)$ represents the current's in-phase, dissipative response at frequency ω to the driving field at frequency ω, and $\sigma_2(\omega)$ represents the current's out-of-phase, non-dissipative response at the same frequency [22].

If the thickness of the film d is much less than a wavelength of the incident radiation, then the right-hand side of Faraday's law applied to the loop shown in Figure 2 encloses a negligible amount of magnetic flux Φ_B, so that

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt} \to 0 \ . \tag{6}$$

Using the sign conventions introduced above, one finds that

$$1 - r - t = 0 \ . \tag{7}$$

Now let us apply Ampere's law [23]

$$\oint \mathbf{H} \cdot d\mathbf{l} = I \tag{8}$$

to the "Amperian" loop (not shown in Figure 2) whose plane is parallel to the magnetic fields of the incident, reflected, and transmitted EM waves, and perpendicular to the Faraday's law loop shown in Figure 2. Let this Amperian loop span the entire width w of the film in the direction of the magnetic field. For a plane EM wave propagating in free space,

$$|\mathbf{H}| = \frac{|\mathbf{B}|}{\mu_0} = \frac{|\mathbf{E}|}{Z_0} \ , \tag{9}$$

where Z_0 is the characteristic impedance of free space and μ_0 is the magnetic permeability of free space. It then follows that

$$w(1 + r - t)\frac{E_{\text{incident}}}{Z_0} = I \ , \tag{10}$$

where

$$I = Aj = \sigma w d(1 - r)E_{\text{incident}} \tag{11}$$

is the total enclosed current being driven inside the film by the applied electric field inside the film (4), which leads to

$$\frac{1}{Z_0}\,(1 + r - t) = \sigma d(1 - r) \ . \tag{12}$$

From (7) and (12) we have two equations in the two unknowns r and t, which can be rewritten as

$$1 - r - t = 0 \ , \text{ and} \tag{13a}$$

$$1 + r - t = x(1 - r) \ , \tag{13b}$$

where $x \equiv \sigma Z_0 d$. Solving for $1/t$ and $1/r$, one obtains

$$\frac{1}{t} = 1 + \frac{1}{2}x, \text{ and} \tag{14a}$$

$$\frac{1}{r} = 1 + 2\frac{1}{x}. \tag{14b}$$

Using the definition $\mathcal{T} = tt^* = |t|^2$, one then obtains for the reciprocal of the transmissivity

$$\begin{aligned}\frac{1}{\mathcal{T}} = \frac{1}{tt^*} &= \left(1 + \frac{1}{2}x\right)\left(1 + \frac{1}{2}x^*\right) \\ &= 1 + \frac{1}{2}(x + x^*) + \frac{1}{4}xx^* \\ &= 1 + \operatorname{Re} x + \frac{1}{4}\left\{(\operatorname{Re} x)^2 + (\operatorname{Im} x)^2\right\} \\ &= \left(1 + \frac{1}{2}\operatorname{Re} x\right)^2 + \frac{1}{4}(\operatorname{Im} x)^2 \ .\end{aligned} \tag{15}$$

Substituting $x = \sigma Z_0 d = (\sigma_1 + i\sigma_2)Z_0 d$ into this expression, one finds that

$$\mathcal{T} = \left\{\left(1 + \frac{1}{2}\sigma_1 Z_0 d\right)^2 + \left(\frac{1}{2}\sigma_2 Z_0 d\right)^2\right\}^{-1} \ . \tag{16}$$

This general result, which applies to *any* thin metallic film with a complex conductivity, agrees with Tinkham's expression for $\mathcal{T}$ [2, Eq. (3.128)] in the case of a superconducting film when the index of refraction of the film's substrate in his expression is set equal to unity (i.e., when the film is surrounded on *both* sides by free space).

Similarly, using the definition $\mathcal{R} = rr^* = |r|^2$, one obtains for the reciprocal of the reflectivity

$$\begin{aligned}\frac{1}{\mathcal{R}} = \frac{1}{rr^*} &= (1 + 2y)(1 + 2y^*) \\ &= 1 + 2(y + y^*) + 4yy^* \\ &= 1 + 4\operatorname{Re} y + 4\left\{(\operatorname{Re} y)^2 + (\operatorname{Im} y)^2\right\} \\ &= (1 + 2\operatorname{Re} y)^2 + 4(\operatorname{Im} y)^2 \ ,\end{aligned} \tag{17}$$

where

$$y \equiv \frac{1}{x} = \frac{1}{\sigma Z_0 d} = \frac{\rho}{Z_0 d} \tag{18}$$

and ρ is the complex resistivity of the film (again at frequency ω). In general, ρ and σ are related by

$$\rho \equiv \frac{1}{\sigma} = \frac{1}{\sigma_1 + i\sigma_2} = \frac{\sigma_1 - i\sigma_2}{\sigma_1^2 + \sigma_2^2} = \rho_1 + i\rho_2 \ , \tag{19}$$

where

$$\rho_1 = \frac{\sigma_1}{\sigma_1^2 + \sigma_2^2} \tag{20a}$$

$$\rho_2 = -\frac{\sigma_2}{\sigma_1^2 + \sigma_2^2} \ . \tag{20b}$$

The reflectivity of any thin metallic film with complex conductivity is therefore

$$\mathcal{R} = \left\{ \left(1 + 2\frac{\sigma_1}{\sigma_1^2 + \sigma_2^2}\frac{1}{Z_0 d}\right)^2 + \left(2\frac{\sigma_2}{\sigma_1^2 + \sigma_2^2}\frac{1}{Z_0 d}\right)^2 \right\}^{-1} . \tag{21}$$

Although the precise degree of reflection for a film of given thickness d will depend on the specific character of the film's conductivity, the presence of the sum inside the first squared term of (21) indicates that the dissipative component of the conductivity σ_1 will inhibit reflection more strongly than the non-dissipative component σ_2. With this clear hint of the importance of dissipationlessness for achieving specular reflection, we turn our attention to superconducting films.

IV. A CRITERION FOR THE SPECULAR REFLECTION OF EM WAVES FROM SUPERCONDUCTING FILMS

The BCS theory of superconductivity has been confirmed by many experiments. Here we review the application of this well-established theory to the problem of mirror-like reflection of EM waves from a superconducting film. We consider once again a film whose thickness d is small enough to make the use of "lumped-circuit" concepts legitimate, but which is now also much smaller than the coherence length ξ_0 and the London penetration depth λ_{L} of the material (i.e., the so-called "local" or "dirty" limit).

As Tinkham has noted [2, p. 39], the dissipative part of the conductivity of such a film $\sigma_{1\mathrm{s}}$ goes *exponentially* to zero as $T \to 0$ in response to a driving wave whose frequency is less than

$$\omega_{\mathrm{gap}} = \frac{2\Delta(0)}{\hbar} \cong \frac{3.5 k_{\mathrm{B}} T_{\mathrm{c}}}{\hbar} , \tag{22}$$

where $\Delta(0)$ (henceforward abbreviated as Δ) is the gap energy per electron of the BCS theory at $T = 0$, k_{B} is Boltzmann's constant, and T_{c} is critical temperature for the superconducting transition. The exponential suppression of the film's dissipative response is due to the "freezing out" of its normal electrons through the Boltzmann factor $\exp(-\Delta/k_{\mathrm{B}}T)$ as $T \to 0$.

On the other hand, the film's non-dissipative conductivity $\sigma_{2\mathrm{s}}$ rises asymptotically to some finite value in the same limit [2, Eq. (3.125)]. The behavior of $\sigma_{2\mathrm{s}}$, which can be calculated using the BCS theory, is due to the film's inductive reactance X_{L}, which in turn arises from its inductance (per square element of the film) L. These three parameters are related to one another by

$$\frac{1}{\sigma_{2\mathrm{s}} d} = X_{\mathrm{L}} = \omega L . \tag{23}$$

For a superconducting film at temperatures sufficiently near $T = 0$ (e.g., in the milli-Kelvin range for a Pb film) and for frequencies lower than $\omega_{\rm gap}$, the ohmic dissipation of the film will be exponentially suppressed by the Boltzmann factor, so that one can, to a good approximation, set $\sigma_{1\rm s} = 0$ and rewrite (21) as [24]:

$$\mathcal{R}_{\rm s} = \left\{ 1 + \left(2 \frac{X_{\rm L}}{Z_0} \right)^2 \right\}^{-1} . \tag{24}$$

The two previous expressions allow us to define an "upper roll-off frequency" $\omega_{\rm r}$ for the reflection of EM waves from a superconducting film, i.e., the frequency at which reflectivity drops to 50% (when the film is kept at nearly $T = 0$ and when $\omega < \omega_{\rm gap}$):

$$\omega_{\rm r} = \pm \frac{Z_0}{2L} , \tag{25}$$

where we discard the negative solution as being unphysical. The film's lower roll-off frequency is simply determined by its lateral dimensions, which for mirror-like behavior to occur must be, as noted before, much larger than the wavelength $\lambda = (2\pi c)/\omega$ of the incident EM wave. Because the upper roll-off frequency is our primary concern, we refer to it throughout as *the* roll-off frequency. Unlike the lower roll-off frequency, it depends on the intrinsic properties of the material and cannot be adjusted at will by altering the lateral dimensions of the film.

The physical meaning of the expression for $\omega_{\rm r}$ given in (25) is that a superconducting film whose dissipative conductivity has been exponentially frozen out can "short out" and thus specularly reflect an incoming EM wave whose frequency is below $\omega_{\rm gap}$ *as long as the film's inductance is sufficiently small to allow non-dissipative supercurrents to flow at frequencies less than* $\omega_{\rm gap}$. As happens with an RF choke, a large inductance will prevent supercurrents from being established inside the film. Thus, the roll-off frequency and reflectivity will be lowered to levels on par with those of a normal metal.

From (25) it is clear that the possibility of specular reflection of EM waves by a superconducting film at low temperatures and frequencies depends crucially on the film's inductance L. The inductance will have two components: a magnetic inductance $L_{\rm m}$ due to the magnetic fields created by the charge supercurrents carried by the Cooper pairs, and a kinetic inductance $L_{\rm k}$ due to the inertial mass of the same Cooper pairs, which causes them to oppose the accelerating force of the external electric field [2, pp. 88, 99][8]. As it happens, $L_{\rm m}$ is numerically negligible compared to $L_{\rm k}$ for a thin film (see Appendix A), so that we can proceed under the assumption that $L \cong L_{\rm k}$.

When $T \ll T_{\rm c}$ and $\omega \ll \omega_{\rm gap}$, the BCS theory yields the following relation between the imaginary part of a superconducting film's complex conductivity $\sigma_{2\rm s}$ and its normal conductivity $\sigma_{\rm n}$ [2, Eq.

(3.125a)]:

$$\sigma_{2\mathrm{s}} = \frac{\pi\Delta}{\hbar\omega}\sigma_{\mathrm{n}} \ . \tag{26}$$

From the Drude model of metallic conductivity, it follows [25] that a film of thickness d will have a normal conductivity σ_{n} given by

$$\sigma_{\mathrm{n}} = \frac{n_{\mathrm{e}} e^2 d}{m_{\mathrm{e}} v_{\mathrm{F}}} \ , \tag{27}$$

where e is the charge of the electron, m_{e} is its mass, v_{F} is its Fermi velocity, and n_{e} is the number density of conduction electrons. Then $\sigma_{2\mathrm{s}}$ becomes

$$\sigma_{2\mathrm{s}} = \frac{\pi\Delta}{\hbar\omega} \cdot \frac{n_{\mathrm{e}} e^2 d}{m_{\mathrm{e}} v_{\mathrm{F}}} \ , \tag{28}$$

from which it follows that the kinetic inductance can be expressed as

$$L_{\mathrm{k}} = \frac{1}{\omega\sigma_{2\mathrm{s}} d} = \frac{1}{d^2} \cdot \frac{\hbar v_{\mathrm{F}}}{\pi\Delta} \cdot \frac{m_{\mathrm{e}}}{n_{\mathrm{e}} e^2} \ . \tag{29}$$

The $1/d^2$ term in (29) indicates a dependence on the film's thickness, whereas the presence of $\hbar v_{\mathrm{F}}/\pi\Delta$ implies an additional dependence on the coherence length ξ_0, since according to the BCS theory

$$\xi_0 = \frac{\hbar v_{\mathrm{F}}}{\pi\Delta} \ . \tag{30}$$

The $m_{\mathrm{e}}/n_{\mathrm{e}} e^2$ term could be interpreted as the London penetration depth λ_{L}, since

$$\mu_0 \lambda_{\mathrm{L}}^2 = \frac{m_{\mathrm{e}}}{n_{\mathrm{e}} e^2} \ . \tag{31}$$

However, in the present context it is more appropriate to relate this term to the plasma frequency ω_{p} by

$$\mu_0 \frac{c^2}{\omega_{\mathrm{p}}^2} = \frac{m_{\mathrm{e}}}{n_{\mathrm{e}} e^2} \ , \tag{32}$$

since the Cooper pairs within a superconductor can be regarded as a type of quantum mechanical, collisionless plasma [26]. We are, after all, concerned not with the screening of DC magnetic fields through the Meissner effect, but with the reflection of EM radiation – with an *electrodynamic* effect rather than a *magnetostatic* one. In the limit of $\omega \ll \omega_{\mathrm{p}}$, the plasma skin depth δ_{p} (the depth to which an EM wave with a frequency ω can penetrate into a plasma) is simply

$$\delta_{\mathrm{p}} = \frac{c}{\omega_{\mathrm{p}}} \ , \tag{33}$$

so that in this limit

$$\mu_0 \delta_\mathrm{p}^2 = \frac{m_\mathrm{e}}{n_\mathrm{e} e^2} \ . \tag{34}$$

Comparing (34) with (31), we see that the electrodynamic concept of the plasma skin depth and the magnetostatic limit given by the London penetration depth coincide not just in the stronger limit of $\omega \to 0$ but also in the weaker limit of $\omega \ll \omega_\mathrm{p}$.

In light of these considerations, we can re-express the kinetic inductance L_k (29) in terms of the permeability of free space μ_0, the coherence length ξ_0, the plasma skin depth δ_p, and the thickness of the film d:

$$L_\mathrm{k} = \mu_0 \xi_0 \left(\frac{\delta_\mathrm{p}}{d} \right)^2 \ . \tag{35}$$

It is then possible to express L_k in more familiar form, i.e., as the product of the magnetic permeability of free space and the kinetic inductance length scale l_k:

$$L_\mathrm{k} = \mu_0 l_\mathrm{k} \ , \tag{36}$$

where l_k is

$$l_\mathrm{k} = \xi_0 \left(\frac{\delta_\mathrm{p}}{d} \right)^2 \ . \tag{37}$$

(For a comparison of this BCS-based derivation of l_k with one based on plasma concepts, see Appendix B.)

We can now rewrite the film's inductive reactance X_L in terms of the frequency of the incident EM wave ω, the permeability of free space μ_0, and the kinetic inductance length scale l_k:

$$X_\mathrm{L} = \omega L_\mathrm{k} = \omega \mu_0 l_\mathrm{k} \ . \tag{38}$$

Returning to the crucial ratio of the inductive reactance to the characteristic impedance of free space given earlier in (25), we see that the roll-off frequency becomes

$$\omega_\mathrm{r} = \frac{Z_0}{2L_\mathrm{k}} = \frac{\mu_0 c}{2\mu_0 l_\mathrm{k}} = \frac{c}{2 l_\mathrm{k}} \ . \tag{39}$$

Notice that μ_0 cancels out of the numerator and denominator of this expression, so that the specular reflection of an EM wave with frequency ω from a superconducting film at temperatures sufficiently near $T = 0$ depends only on the ratio of the speed of light c to the kinetic inductance length scale l_k.

To make this claim concrete, let us consider here (and in subsequent examples) the case of a thin lead (Pb) film with a thickness of $d = 2$ nm and an angular frequency for the incident radiation

of $\omega = 2\pi \times$ (6 GHz). The known values for the coherence length and the London penetration depth of Pb are $\xi_0 = 83$ nm and $\delta_\mathrm{p} = \lambda_\mathrm{L} = 37$ nm, respectively [27, p. 24]. Inserting these values into (37), we see that $l_\mathrm{k} \approx 30$ μm and, from (39), that $\omega_\mathrm{r} \approx 2\pi \times$ (800 GHz). When we recall that the theoretically calculated gap frequency for superconducting Pb at $T = 0$ is approximately $2\pi \times$ (500 GHz), we see that our estimate of ω_r is roughly equivalent to the claim that $\omega < \omega_\mathrm{gap}$ for specular reflection to occur, which is consistent with previously stated assumptions (and with the requirement that $\omega \ll \omega_\mathrm{p}$, since $\omega_\mathrm{p} \approx 2\pi\times$ (1.3 PHz) for Pb).

The analysis presented in this section is in basic agreement with the experiments of Glover and Tinkham [1], and it belies the commonly held misconception that specular reflection can occur only when the thickness of the material d is greater than its skin depth δ_p (or penetration depth λ_L). Reflection from a superconducting film is due not to the gradual diminishment of the radiation field as it enters the film but to the destructive interference between the incident radiation and the radiation emitted in the forward scattering direction by the sheet supercurrents set up within the film. In fact, a closer examination of (37) and (39) reveals that appreciable reflection of a 6 GHz EM wave can occur from a Pb film – a type I superconductor – even when the film's thickness is as much as 2 orders of magnitude smaller than its characteristic penetration depth. A type II superconductor, on the other hand, will generate considerable losses, due to the ohmic or dissipative flux-flow motion of Abrikosov vortices at microwave frequencies, and will therefore exhibit much poorer reflectivities in the microwave region.

What does the foregoing analysis imply about the ability of a superconducting film to reflect a GR microwave? In order to answer this question we must determine the magnitude of the kinetic inductance length scale in the GR case. First, however, we will take a moment to motivate the idea of the "characteristic gravitational impedance of free space" and to consider why objects made of normal matter are such poor reflectors of GR waves.

V. THE GRAVITATIONAL CHARACTERISTIC IMPEDANCE OF FREE SPACE

Wald [28, Section 4.4] has introduced an approximation scheme that leads to a useful Maxwell-like representation of the Einstein equations of general relativity. The resulting equations describe the coupling of weak GR fields to slowly moving matter. In the asymptotically flat spacetime

coordinate system of a distant inertial observer, the four equations in SI units are

$$\nabla \cdot \mathbf{E}_{\mathrm{G}} = -\frac{\rho_{\mathrm{G}}}{\varepsilon_{\mathrm{G}}} \tag{40a}$$

$$\nabla \times \mathbf{E}_{\mathrm{G}} = -\frac{\partial \mathbf{B}_{\mathrm{G}}}{\partial t} \tag{40b}$$

$$\nabla \cdot \mathbf{B}_{\mathrm{G}} = 0 \tag{40c}$$

$$\nabla \times \mathbf{B}_{\mathrm{G}} = \mu_{\mathrm{G}} \left(-\mathbf{j}_{\mathrm{G}} + \varepsilon_{\mathrm{G}} \frac{\partial \mathbf{E}_{\mathrm{G}}}{\partial t} \right) \tag{40d}$$

where the gravitational analog of the electric permittivity of free space is given by

$$\varepsilon_{\mathrm{G}} = \frac{1}{4\pi G} = 1.2 \times 10^{9} \text{ SI units} \tag{41}$$

and the gravitational analog of the magnetic permeability of free space is given by

$$\mu_{\mathrm{G}} = \frac{4\pi G}{c^2} = 9.3 \times 10^{-27} \text{ SI units.} \tag{42}$$

The value of ε_{G} is fixed by demanding that Newton's law of gravitation be recovered from the Gauss-like law (40a), whereas the value of μ_{G} is fixed by the linearization procedure from Einstein's field equations. These two constants express the strengths of the coupling between sources (i.e., of masses and mass currents, respectively) and gravitational fields, and are analogous to the two constants ε_0 (the permittivity of free space) and μ_0 (the permeability of free space), which express the strengths of coupling between sources (charges and charge currents, respectively) and electromagnetic fields in Maxwell's theory.

In the above set of equations, the field $\mathbf{E}_{\mathrm{G}}$ is the *gravito*-electric field, which is to be identified with the local acceleration $\mathbf{g}$ of a test particle produced by the mass density ρ_{G}, in the Newtonian limit of general relativity. The field $\mathbf{B}_{\mathrm{G}}$ is the *gravito*-magnetic field produced by the mass current density $\mathbf{j}_{\mathrm{G}}$ and by the gravitational analog of the Maxwell displacement current density $\varepsilon_{\mathrm{G}}\partial\mathbf{E}_{\mathrm{G}}/\partial t$ of the Ampere-like law (40d). The resulting magnetic-like field $\mathbf{B}_{\mathrm{G}}$ can be regarded as a generalization of the Lense-Thirring field of general relativity. Because these equations are linear, all fields will obey the superposition principle not only outside the source (i.e., in the vacuum), but also within the matter inside the source, provided the field strengths are sufficiently weak and the matter is sufficiently slowly moving. Note that the fields $\mathbf{E}_{\mathrm{G}}$ and $\mathbf{B}_{\mathrm{G}}$ in the above Maxwell-like equations will be treated as *classical* fields, just like the fields $\mathbf{E}$ and $\mathbf{B}$ in the classical Maxwell's equations.

As noted earlier, Cooper pairs cannot freely fall along with the ionic lattice in response to an incident GR wave because the UP forbids such pairs from having classical trajectories, i.e., from traveling along geodesics. An incident field $\mathbf{E}_{\mathrm{G}}$ will therefore cause the Cooper pairs to undergo *non-geodesic* motion, in contrast to the *geodesic* motion of the ions inside the lattice. This entails

the existence of mass currents (as well as charge currents) from the perspective of a local, freely falling observer who is located near the surface of the superconducting film anywhere other than at its center of mass. These mass currents will be describable by a gravitational version of Ohm's law

$$\mathbf{j}_{\mathrm{G}}(\omega)=\sigma_{\mathrm{s,G}}(\omega)\mathbf{E}_{\text{G-inside}}(\omega)\ , \tag{43}$$

where $\mathbf{j}_{\mathrm{G}}(\omega)$ is the mass-current density at frequency ω, $\sigma_{\mathrm{s,G}}(\omega)=\sigma_{1\mathrm{s,G}}(\omega)+i\sigma_{2\mathrm{s,G}}(\omega)$ is the complex mass-current conductivity of the film at the frequency ω in its linear response to the fields of the incident GR wave, and $\mathbf{E}_{\text{G-inside}}(\omega)$ is the driving gravito-electric field inside the film at frequency ω. The existence of these mass currents can also be inferred from DeWitt's minimal coupling rule for superconductors ([29]; see Section 7 below). The real part of the mass conductivity, $\sigma_{1\mathrm{s,G}}(\omega)$, describes the superconductor's dissipative response to the incident gravito-electric field, while the imaginary part, $\sigma_{2\mathrm{s,G}}(\omega)$, describes its non-dissipative response to the same field. The basic assumption behind (43) is that the mass-current density in any superconductor responds *linearly* to a weak GR wave at the driving frequency [30]. One should view $\sigma_{\mathrm{s,G}}$ as a phenomenological quantity, which, like the electrical conductivity σ_{s}, must be experimentally determined. In any case, the resulting optics for weak GR waves will be *linear*, just like the linear optics for weak EM waves.

An important physical property follows from the above Maxwell-like equations, namely, the characteristic gravitational impedance of free space Z_{G} [18, 31, 32]:

$$Z_{\mathrm{G}}=\sqrt{\frac{\mu_{\mathrm{G}}}{\varepsilon_{\mathrm{G}}}}=\frac{4\pi G}{c}=2.8\times10^{-18}\ \text{SI units.} \tag{44}$$

This quantity is a characteristic of the vacuum, i.e., it is a property of spacetime itself, and it is independent of any of the properties of matter *per se*. As with $Z_0=\sqrt{\mu_0/\varepsilon_0}=377$ ohms in the EM case, $Z_{\mathrm{G}}=\sqrt{\mu_{\mathrm{G}}/\varepsilon_{\mathrm{G}}}=2.8\times10^{-18}$ SI units will play a central role in all GR radiation coupling problems. In practice, the impedance of a material object must be much smaller than this extremely small quantity before any significant portion of the incident GR-wave power can be reflected. In other words, conditions must be highly unfavorable for dissipation into heat. Because all classical material objects have extremely high levels of dissipation compared to Z_{G}, even at very low temperatures, they are inevitably very poor reflectors of GR waves [21, 32]. The question of GR-wave reflection from macroscopically coherent quantum systems such as superconductors requires a separate analysis due to the effectively zero resistance associated with superconductors, i.e., the dissipationlessness exhibited by matter in this unique state, at temperatures near absolute zero.

VI. A CRITERION FOR THE SPECULAR REFLECTION OF GR WAVES FROM SUPERCONDUCTING FILMS

In the case of EM waves considered above in Section 4, the BCS framework led us to two related expressions for the behavior of a superconducting thin film, one for its EM reflectivity (24) and one for its EM roll-off frequency (25). Now, on the basis of the similarity of the Maxwell and the Maxwell-like equations, the identicality of the boundary conditions that follow from these equations, and the linearity of weak GR-wave optics that follows from the gravitational version of Ohm's law for superconductors (43), we are led to the following two expressions for the reflectivity and the roll-off frequency in the GR sector, which are analogous to (24) and (25), respectively:

$$\mathcal{R}_{\mathrm{G}} = \left\{ 1 + \left(2\frac{X_{\mathrm{L,G}}}{Z_{\mathrm{G}}} \right)^2 \right\}^{-1} \tag{45a}$$

$$\omega_{\mathrm{r,G}} = \pm \frac{Z_{\mathrm{G}}}{2L_{\mathrm{G}}} \ . \tag{45b}$$

Once again, we exclude the negative solution in the expression for the upper roll-off frequency given in (45b) as being unphysical.

Pausing for a moment to consider the *lower* roll-off frequency, we find that a new constraint appears. The H-C effect, which is ultimately responsible for the mirror-like behavior of the film in the GR case, can only be presumed to operate when

$$\omega \geq \frac{2\pi v_{\mathrm{s}}}{a} \ , \tag{46}$$

where ω is the frequency of the incident wave, v_s is the speed of sound in the medium, and a is the transverse size of a square film. The physical significance of this constraint becomes apparent when we rewrite it as

$$a \geq \frac{2\pi v_{\mathrm{s}}}{\omega} \ . \tag{47}$$

This form of the inequality follows from the fact that neighboring ions separated by a distance less than $(2\pi v_{\mathrm{s}})/\omega$ will be mechanically coupled to one another, since there will be sufficient time for a mechanical signal to propagate from one to the other. Only ions separated by distances greater than $(2\pi v_{\mathrm{s}})/\omega$ can be legitimately regarded as separately undergoing free fall in the presence of a GR wave. Ultimately, however, this additional constraint is preempted by the inequality already introduced in Section 2,

$$a \geq \frac{2\pi c}{\omega} \ , \tag{48}$$

where $(2\pi c)/\omega$ is the wavelength of the incident wave, since this more stringent requirement must be met for the film to function as a mirror at all.

Returning to the expression for the *upper* roll-off frequency given in (45b), is it conceivable that this expression could yield a non-negligible $\omega_{\mathrm{r,G}}$ in the case of a superconducting film? We begin by noting that the gravitational impedance of free space Z_{G} can be expressed as

$$Z_{\mathrm{G}} = \mu_{\mathrm{G}} c \ . \tag{49}$$

In light of (45b) and the smallness of μ_{G}, as indicated earlier in (42), it would seem highly unlikely that a superconductor's GR inductance would be small enough to produce a non-negligible roll-off frequency. Any attempt to construct laboratory-scale mirrors for GR waves would appear to be doomed from the start. However, as with L for a thin film in the electromagnetic case, L_{G} must be expressible as the product of the permeability and a length scale. In the GR case, we must use the *gravitational* version of each parameter. We will neglect the contribution of the gravito-magnetic inductance $L_{\mathrm{m,G}}$ to the overall gravitational inductance L_{G} on the grounds that it will be much smaller than the gravito-kinetic inductance $L_{\mathrm{k,G}}$ (again, see Appendix A), so that

$$L_{\mathrm{G}} \approx L_{\mathrm{k,G}} = \mu_{\mathrm{G}} l_{\mathrm{k,G}} \ . \tag{50}$$

Inserting (49) and (50) into (45b), we see that the permeability cancels out of the numerator and denominator as before, so that $\omega_{\mathrm{r,G}}$ depends only on the ratio of the speed of light c to a single parameter – in this case, the *gravitational* kinetic inductance length scale $l_{\mathrm{k,G}}$:

$$\omega_{\mathrm{r,G}} = \frac{\mu_{\mathrm{G}} c}{2\mu_{\mathrm{G}} l_{\mathrm{k,G}}} = \frac{c}{2 l_{\mathrm{k,G}}} \ . \tag{51}$$

In the electromagnetic case, l_{k} was given by

$$l_{\mathrm{k}} = \xi_0 \left(\frac{\delta_{\mathrm{p}}}{d} \right)^2 \ , \tag{52}$$

where the plasma skin depth δ_{p} was given by

$$\delta_{\mathrm{p}} = \sqrt{\frac{m_{\mathrm{e}}}{\mu_0 n_{\mathrm{e}} e^2}} \ . \tag{53}$$

In the present context, the coherence length ξ_0 and the thickness of the film d must remain the same, since they are *internal* properties of the film having nothing to do with the strength of coupling to *external* radiation fields. By contrast, the plasma skin depth *would* appear to depend on the strength of coupling to external radiation fields through the presence of μ_0 and e^2 in the denominator of (53). We therefore need to consider the magnitude of this parameter in the gravitational sector.

For the moment, let us assume that the coupling of Cooper pairs to a GR wave depends *solely* on their gravitational mass $2m_{\mathrm{e}}$, i.e., that their electrical charge $2e$ is irrelevant to the gravitational plasma skin depth and thus to the gravitational kinetic inductance length scale of a superconducting

film. Ultimately, we will reject this approach, since the Coulomb interaction between the superconductor's Cooper pairs and the corresponding holes created in the virtual plasma excitation induced within the film is crucial for understanding how the film responds to a GR wave. Nonetheless, it is instructive to ignore all considerations of charge and to presume, for the moment, that Cooper pairs react to a GR wave solely on the basis of their mass. In fact, the criterion presented at the end of this section may well be valid for neutral superfluids (e.g., superfluid helium or a neutral atomic Bose-Einstein condensate), but we show in the following section that it must be modified in the case of superconductors to account for the H-C effect.

To obtain the "gravitational" version of the plasma skin depth $\delta_{\mathrm{p,G}}$, let us make the following substitution

$$\frac{e^2}{4\pi\varepsilon_0} \rightarrow Gm_{\mathrm{e}}^2 \tag{54}$$

in the expression for the plasma skin depth δ_{p} (53). Note that for this substitution to be valid, we must treat the electrons as if they were electrically neutral. The "gravitational" kinetic inductance length scale then becomes

$$l_{\mathrm{k,G}} = \xi_0 \left(\frac{\delta_{\mathrm{p,G}}}{d} \right)^2 , \tag{55}$$

where, in this spurious approach, $\delta_{\mathrm{p,G}}$ is given by

$$\delta_{\mathrm{p,G}} = \sqrt{\frac{1}{\mu_{\mathrm{G}} n_{\mathrm{e}} m_{\mathrm{e}}}} . \tag{56}$$

Assuming here and in subsequent calculations an estimate of $n = n_{\mathrm{e}}/2 \approx 10^{30}$ m^{-3} for the number density of Cooper pairs, one finds that $\delta_{\mathrm{p,G}}$ is on the order of 10^{13} m, which leads to a value for $l_{\mathrm{k,G}}$ on the order of 10^{36} m. Inserting this enormous value for $l_{\mathrm{k,G}}$ into (51) yields a roll-off frequency $\omega_{\mathrm{r,G}}$ of effectively zero, which of course undermines any practical possibility of GR-wave reflection.

On the grounds that one must eliminate dissipation into heat for the GR-wave scattering cross-section to become comparable to a square wavelength, Weinberg has suggested in his discussion of Weber-style resonant bar detectors that superfluids might function effectively as mirrors for GR waves [21]. The analysis presented here, however, suggests that neutral superfluids cannot substantially reflect GR waves because of the electrical neutrality of their mass carriers. (See Appendix C for a brief account of the relation between the "impedance" argument of the previous section and Weinberg's analysis of the dissipation problem.) As we shall see, the fact that a superconductor's mass carriers are not electrically neutral utterly changes the dynamics of the interaction.

VII. THE SPECULAR REFLECTION OF GR WAVES

In Section 2, we argued that the Uncertainty Principle *delocalizes* a superconductor's Cooper pairs within the material, so that they must exhibit *non-geodesic* motion rather than the decoherence-induced *geodesic* motion exhibited by all localized particles, such as freely floating "dust particles" or the ions in the lattice of a superconductor. The non-localizability of the Cooper pairs within a superconducting film leads to charge supercurrents inside the film, which, by charge conservation and the accumulation of charge at its edges, must produce a Coulomb electric field inside the film in a virtual plasma excitation of the material. As a result, enormous Coulomb forces will be created between the film's negatively charged Cooper pairs and its corresponding, positively charged holes.

In the GR case, one might think to replace the Coulomb force with the much weaker Newtonian gravitational force – as we did in the previous section – but this amounts to treating the Cooper pairs and holes as if they were electrically neutral, which is patently unphysical. There can be no H-C effect in the case of a neutral superfluid, but the situation is entirely different for a superconductor. This effect, which can appear inside a superconductor, causes a superconducting film to respond extremely "stiffly" to an incident GR wave and leads to hard-wall boundary conditions for the wave. To put the point differently, the stiffness of a superconducting film in its response to an incoming GR wave is governed by the strength of the Coulomb interaction between the Cooper pairs and the corresponding holes, and not by their much weaker gravitational interaction. This fact is reflected in the appearance of the electromagnetic plasma frequency in the formulas derived below.

Let us begin our analysis of the magnitude of the H-C effect by examining the quantum probability current density $\mathbf{j}$. This quantity is more basic than the charge current density $\mathbf{j}_{\rm e} = nq\mathbf{v}$ or the mass current density $\mathbf{j}_{\rm G} = nm\mathbf{v}$, since one can derive $\mathbf{j}$ directly from quantum mechanics. It should be regarded as the *cause* of the charge and mass currents, whereas $\mathbf{j}_{\rm e}$ and $\mathbf{j}_{\rm G}$ should be regarded as the *effects* of $\mathbf{j}$.

Recall that in non-relativistic quantum mechanics $\mathbf{j}$ is given by

$$\mathbf{j} = \frac{\hbar}{2mi}(\psi^* \nabla \psi - \psi \nabla \psi^*) \ , \tag{57}$$

where m is the mass of the non-relativistic particle whose current is being calculated (here $m = 2m_e$) and ψ is the wavefunction of the system (here the Cooper pair's "condensate wavefunction", or London's "macroscopic wavefunction", or Ginzburg and Landau's "complex order parameter"). This quantum mechanical quantity satisfies the continuity equation

$$\nabla \cdot \mathbf{j} + \frac{\partial \rho}{\partial t} = 0 \ , \tag{58}$$

where $\rho = \psi^*\psi$ is the quantum probability density of the Cooper pairs. The meaning of (58) is that probability is conserved.

Now let us adopt DeWitt's minimal coupling rule [29] and make the following substitution for the momentum operator:

$$\mathbf{p} \to \mathbf{p} - q\mathbf{A} - m\mathbf{h} \text{ or} \tag{59a}$$

$$\frac{\hbar}{i}\nabla \to \frac{\hbar}{i}\nabla - q\mathbf{A} - m\mathbf{h} \ , \tag{59b}$$

where $q = 2e$, $m = 2m_e$, $\mathbf{A}$ is the electromagnetic vector potential, and $\mathbf{h}$ is DeWitt's gravitational vector potential [33] (here and henceforth the dependence on space and time $(\mathbf{r}, t)$ of all field quantities will be suppressed as understood). In what follows, both $\mathbf{A}$ and $\mathbf{h}$ fields will be treated as *classical* fields, whereas $\mathbf{j}$ and ρ will be treated as time-dependent *quantum* operators, in a *semi-classical* treatment of the interaction of radiation with matter.

We shall also follow DeWitt in adopting the radiation gauge conditions for both $\mathbf{A}$ and $\mathbf{h}$, namely, that

$$\nabla \cdot \mathbf{A} = 0 \text{ and } \nabla \cdot \mathbf{h} = 0 \ , \tag{60}$$

and that the scalar potentials for both the EM and GR fields vanish identically everywhere. This choice of gauge means that the coordinate system being employed is that of an inertial observer located at infinity.

Since it is the case that

$$\frac{\hbar}{2mi}(\psi^*\nabla\psi - \psi\nabla\psi^*) = \frac{1}{m}\,\mathrm{Re}\left(\psi^*\frac{\hbar}{i}\nabla\psi\right) \ , \tag{61}$$

we can apply DeWitt's minimal coupling rule to (61) to obtain

$$\mathbf{j} = \frac{1}{m}\,\mathrm{Re}\left(\psi^*\left\{\frac{\hbar}{i}\nabla - q\mathbf{A} - m\mathbf{h}\right\}\psi\right) \ . \tag{62}$$

The continuity equation (58) is still satisfied by (62), provided that one also applies the same minimal coupling rule to the time-dependent Schrödinger equation, in which the Hamiltonian becomes

$$H = \frac{(\mathbf{p} - q\mathbf{A} - m\mathbf{h})^2}{2m} + V \ , \tag{63}$$

where the first term on the right-hand side represents the kinetic energy operator, and V is the potential energy operator.

In the special case of *neutral*, classical "dust particles" in the presence of a GR wave, $q = 0$ and thus $q\mathbf{A} = 0$ (as well as $V = 0$). The classical Hamilton's function $H(\mathbf{p}, \mathbf{q})$ then becomes

$$H(\mathbf{p}, \mathbf{q}) = \frac{(\mathbf{p} - m\mathbf{h})^2}{2m} \ . \tag{64}$$

Defining the canonical momentum classically as $\mathbf{p} = m\mathbf{v}_{\mathrm{can}}$, where $\mathbf{v}_{\mathrm{can}}$ is the canonical velocity, it will be the case for neutral, classical dust particles that

$$H = \frac{1}{2}m\left(\mathbf{v}_{\mathrm{can}} - \mathbf{h}\right)^2 = 0 \quad \text{or} \quad \mathbf{v}_{\mathrm{can}} = \mathbf{h}\ , \tag{65}$$

as seen by a distant inertial observer, since a passing GR wave cannot impart any kinetic energy to noninteracting, freely-falling particles. The dust particles will be carried along *with* space, which follows directly from the EP.

On the other hand, when (64) is viewed as a quantum Hamiltonian operator, it implies that neutral, *quantum-mechanical* particles will acquire a kinetic energy equal to $\frac{1}{2}m\mathbf{h}^2$ when they are in a nonlocalizable, gap-protected, zero-momentum eigenstate ($\mathbf{p} = \mathbf{0}$, where $\mathbf{p}$ is the canonical momentum). In accord with first-order time-dependent perturbation theory, such particles must remain in their ground state in the presence of a GR wave whose frequency is less than the BCS gap frequency. They will therefore rigidly resist the stretching and squeezing of space caused by such a wave. In other words, they will be locally accelerated *through* space, acquiring kinetic energy in the process. In the case of superfluid helium, for example, in which the basic components of the material are both electrically neutral and quantum-mechanically protected from excitations by the roton gap, mass supercurrents will be created that carry kinetic energy extracted from the wave.

Now let us consider the case of a type I superconductor. Before the arrival of a GR wave, the superconductor's Cooper pairs will be in a zero-momentum eigenstate:

$$\psi = C\exp\left(i\frac{\mathbf{p}_0 \cdot \mathbf{r}}{\hbar}\right) \text{ where } \mathbf{p}_0 = \mathbf{0}\ . \tag{66}$$

Again, in accord with first-order time-dependent perturbation theory, this initial wavefunction must remain unchanged to lowest order by the radiative perturbations arising from either $\mathbf{A}$ or $\mathbf{h}$ after the arrival of a wave whose frequency is less than the BCS gap frequency of the material. If one evaluates (62) using the unperturbed state (66), one finds that

$$\begin{aligned}\mathbf{j} &= \frac{1}{m}\,\mathrm{Re}\left(\psi^*\left\{\frac{\hbar}{i}\nabla - q\mathbf{A} - m\mathbf{h}\right\}\psi\right) \\ &= \frac{1}{m}\left(-q\mathbf{A} - m\mathbf{h}\right)\psi^*\psi\ .\end{aligned} \tag{67}$$

From this one can define the "quantum velocity field" $\mathbf{v}$,

$$\mathbf{v} = \frac{\mathbf{j}}{\rho} = \frac{\mathbf{j}}{\psi^*\psi}\ , \tag{68}$$

whose local expectation value is the local group velocity of a Cooper pair [34]. It thus follows that

$$\mathbf{v} = -\frac{q}{m}\mathbf{A} - \mathbf{h} \tag{69}$$

inside a superconducting film after the arrival of a GR wave. This velocity is the *kinetic* velocity of the quantum supercurrent, and not the canonical velocity of a classical dust particle given in (65), in the sense that $\frac{1}{2}m\mathbf{v}^2$ is the local kinetic energy of the quantum supercurrent.

The generation of mass supercurrents inside a superconductor by the GR wave will also produce *charge* supercurrents inside the superconductor, since q is not zero for Cooper pairs. These supercurrents will electrically polarize the superconductor, which will set up an *internal* $\mathbf{A}$ field – even in the absence of any incident EM wave. Thus, the term $(-q/m)\mathbf{A}$ on the right-hand side of (69) will *not* be zero inside a superconductor in the presence of a GR wave. Herein lies the possibility of mirror-like reflection of GR waves from superconducting thin films.

Taking the partial derivative of (69) with respect to time, and defining the meaning of this derivative in the sense of Heisenberg's equation of motion for the kinetic velocity operator $\mathbf{v}$, one obtains an operator equation of motion that has the same form as Newton's 2nd law of motion, namely,

$$m\frac{\partial}{\partial t}\mathbf{v} = m\frac{\partial^2}{\partial t^2}\mathbf{x} = m\mathbf{a} = q\mathbf{E} + m\mathbf{E}_{\mathrm{G}} \ , \tag{70}$$

where, by our gauge choice, $\mathbf{E}$ and $\mathbf{E}_{\mathrm{G}}$ inside the superconductor are related to the vector potentials $\mathbf{A}$ and $\mathbf{h}$, respectively, by

$$\mathbf{E} = -\frac{\partial}{\partial t}\mathbf{A} \text{ and } \mathbf{E}_{\mathrm{G}} = -\frac{\partial}{\partial t}\mathbf{h} \ . \tag{71}$$

Both $\mathbf{E}$ and $\mathbf{E}_{\mathrm{G}}$ will be treated here as classical fields. Following the presentation in Section 5, $\mathbf{E}_{\mathrm{G}}$ is the gravito-electric field that appears in the Maxwell-like equations, which is equivalent to the acceleration $\mathbf{g}$ of a local, classical test particle due to gravity, in accord with the EP. The physical interpretation of the Newton-like equation of motion (70) is that the internal $\mathbf{E}$ and $\mathbf{E}_{\mathrm{G}}$ fields act upon the charge q and the mass m, respectively, of the Cooper pairs, to produce an acceleration field $\mathbf{a}$ of these pairs (in the sense of Ehrenfest's theorem) inside a superconducting film.

For all fields that vary sinusoidally with the same exponential phase factor $\exp(-i\omega t)$, (70) leads to the following linear-response equation at the frequency ω:

$$\mathbf{x} = -\frac{1}{\omega^2}\left(\frac{q}{m}\mathbf{E} + \mathbf{E}_{\mathrm{G}}\right) \ . \tag{72}$$

The mass current density source term in the Ampere-like law (40d) of the Maxwell-like equations is then given by

$$\begin{aligned}\mathbf{j}_{\mathrm{G}} &= nm\mathbf{v} = nm\frac{\partial}{\partial t}\mathbf{x} \\ &= nm(-i\omega)\mathbf{x} \\ &= i\frac{n}{\omega}\left(q\mathbf{E}+m\mathbf{E}_{\mathrm{G}}\right) \ .\end{aligned} \tag{73}$$

The total force acting on a given Cooper pair under such circumstances is thus [35]

$$\mathbf{F}_{\text{tot}} = q\mathbf{E} + m\mathbf{E}_{\text{G}} \ , \tag{74}$$

which is to say that $\mathbf{F}_{\text{tot}}$ depends on a *linear combination* of the internal $\mathbf{E}$ and $\mathbf{E}_{\text{G}}$ fields, or, equivalently, that a superconductor will respond *linearly* to a sufficiently weak incident GR wave.

When a superconductor is operating in its *linear response regime* in the presence of a weak incident GR wave, the following direct proportionalities will hold:

$$\mathbf{F}_{\text{tot}} \propto \mathbf{E} \propto \mathbf{E}_{\text{G}} \ . \tag{75}$$

Let us therefore define a proportionality constant Ξ, such that

$$\mathbf{F}_{\text{tot}} = \Xi q\mathbf{E} \ . \tag{76}$$

We shall call this dimensionless proportionality constant the "fractional correction factor" of the total force acting upon a given Cooper pair, relative to a purely electrical force acting on the same pair.

At this point, it would be customary to ignore the extremely weak gravitational forces generated internally within the superconducting film. That is to say, one would normally set the gravitational field $\mathbf{E}_{\text{G}}$ inside the film identically equal to zero everywhere by declaring that $\Xi = 1$, exactly. One could then solve the essentially *electromagnetic* problem of virtual plasma excitations produced inside the film in its linear response to a weak incident EM or GR wave.

But this simplification will not suffice in the present context, since we want to understand the dynamics of the system when one takes into account the *combined* effect of the internal electric field $\mathbf{E}$ and internal gravito-electric field $\mathbf{E}_{\text{G}}$, both of which will be produced in association with the electrical polarization of the superconductor induced by an incident EM or GR wave. Although the impact on the *electro*dynamics of the system will be negligible, the impact on its *gravito*-electrodynamics will be enormous. Let us then use (74) and (76) to express the relationship between the $\mathbf{E}$ and $\mathbf{E}_{\text{G}}$ fields inside a superconducting film when $\Xi \neq 1$, i.e., when the gravitational forces within the film, however tiny, are explicitly taken into account:

$$\mathbf{E} = \frac{1}{\Xi - 1}\frac{m}{q}\mathbf{E}_{\text{G}} \ . \tag{77}$$

Substituting this expression into (73), we obtain [36]

$$\mathbf{j}_{\text{G}} = i\frac{\Xi}{\Xi - 1}\frac{nm}{\omega}\mathbf{E}_{\text{G}} \ , \tag{78}$$

from which it follows that the mass conductivity of the film σ_{G} is given by

$$\sigma_{\text{G}} = i\left(\frac{\Xi}{\Xi - 1}\right)\frac{nm}{\omega} \propto \frac{1}{\omega} \ , \tag{79}$$

implying an inductive response to internal fields on the part of the mass currents $\mathbf{j}_{\mathrm{G}}$ within the film. Note that σ_{G} can in principle become extremely large when $\Xi \rightarrow 1$, and therefore that $\mathbf{j}_{\mathrm{G}}$ can become extremely large.

Let us consider first the effect of the gravitational force between the Cooper pairs and holes on the plasma frequency. We start from (70) in the form

$$m\frac{\partial^2}{\partial t^2}\mathbf{x} = -m\omega^2\mathbf{x} = q\mathbf{E} + m\mathbf{E}_{\mathrm{G}} = \Xi q\mathbf{E} \ , \tag{80}$$

so that

$$\mathbf{x} = -\Xi\frac{q}{m\omega^2}\mathbf{E} \ . \tag{81}$$

The electric polarization of the superconductor will then be

$$\mathbf{P} = nq\mathbf{x} = -\Xi\frac{nq^2}{m\omega^2}\mathbf{E} = \chi_{\mathrm{p}}'\varepsilon_0\mathbf{E} \ , \tag{82}$$

where χ_{p}' is the modified plasma susceptibility. Since this susceptibility can be expressed as

$$\chi_{\mathrm{p}}' = -\frac{\omega_{\mathrm{p}}'^2}{\omega^2} \ , \tag{83}$$

it follows that the square of the modified plasma frequency is given by

$$\omega_{\mathrm{p}}'^2 = \Xi\frac{nq^2}{m\varepsilon_0} \ . \tag{84}$$

We thus expect that the fractional correction factor Ξ, which takes into account the gravitational forces between the Cooper pairs and holes, will lead to an extremely small correction to the standard formula for the plasma frequency.

To determine the magnitude of Ξ, we begin with the quantum form of Newton's second law (70), rewritten as

$$\frac{\partial}{\partial t}\mathbf{v} = \frac{q}{m}\mathbf{E} + \mathbf{E}_{\mathrm{G}} \ . \tag{85}$$

Multiplying both sides by nq, one obtains a current-density form of the same equation:

$$\frac{\partial(nq\mathbf{v})}{\partial t} = \frac{\partial}{\partial t}\mathbf{j}_{\mathrm{e}} = \frac{nq^2\mathbf{E}}{m} + nq\mathbf{E}_{\mathrm{G}} \ . \tag{86}$$

Let us evaluate all quantities in this equation at a point P along the edge of the superconducting film where the ionic lattice abruptly ends and the vacuum begins:

$$\left.\frac{\partial}{\partial t}\mathbf{j}_{\mathrm{e}}\right|_P = \left.\frac{nq^2\mathbf{E}}{m}\right|_P + nq\mathbf{E}_{\mathrm{G}}|_P \ . \tag{87}$$

We will assume that the incident radiation fields that excite the Cooper-pair plasma are tightly focused onto a diffraction-limited Gaussian-beam spot size located at the center of the square film. We will also assume that the radiative excitation is impulsive in nature, so that the plasma can oscillate freely after the radiation is abruptly turned off. Thus the point P at the edge of the film at which all quantities in (87) are to be evaluated, is far away from the center of the film, where the incident radiation fields can impulsively excite the film into free plasma oscillations.

Taking the divergence of both sides of (87), we obtain at point P

$$\frac{\partial}{\partial t}\left(\nabla\cdot\mathbf{j}_{\mathrm{e}}\right)\bigg|_{P}=\frac{nq^{2}}{m}\left(\nabla\cdot\mathbf{E}\right)\bigg|_{P}+nq\left(\nabla\cdot\mathbf{E}_{\mathrm{G}}\right)\big|_{P}. \tag{88}$$

But with the help of the continuity equation

$$\nabla\cdot\mathbf{j}_{\mathrm{e}}+\frac{\partial}{\partial t}\rho_{\mathrm{e}}=0 \tag{89}$$

and the 1st Maxwell and 1st Maxwell-like equations

$$\nabla\cdot\mathbf{E}=\frac{\rho_{\mathrm{e}}}{\varepsilon_{0}}\text{ and }\nabla\cdot\mathbf{E}_{\mathrm{G}}=-\frac{\rho_{\mathrm{G}}}{\varepsilon_{\mathrm{G}}}\ , \tag{90}$$

we can rewrite (88) as a differential equation for the charge and mass densities at point P [37]:

$$-\frac{\partial^{2}}{\partial t^{2}}\rho_{\mathrm{e}}=\frac{nq^{2}}{m\varepsilon_{0}}\rho_{\mathrm{e}}-\frac{nq}{\varepsilon_{\mathrm{G}}}\rho_{\mathrm{G}}. \tag{91}$$

These densities will oscillate freely in time at point P at the edge of the film, where both charge and mass can accumulate, after the impulsive excitation at the center of the film has been turned off. We then use the fact that the accumulated Cooper-pair mass density at point P must be related to the accumulated Cooper-pair charge density at point P by

$$\rho_{\mathrm{G}}=\frac{m}{q}\rho_{\mathrm{e}}\ , \tag{92}$$

since each Cooper pair accumulating at the edge of the film carries with it both a charge q and a mass m. Then at point P (91) becomes

$$-\frac{\partial^{2}}{\partial t^{2}}\rho_{\mathrm{e}}=\frac{nq^{2}}{m\varepsilon_{0}}\rho_{\mathrm{e}}-\frac{nm}{\varepsilon_{\mathrm{G}}}\rho_{\mathrm{e}}\ , \tag{93}$$

which leads to the simple harmonic equation of motion

$$\frac{\partial^{2}}{\partial t^{2}}\rho_{\mathrm{e}}+\frac{nq^{2}}{m\varepsilon_{0}}\rho_{\mathrm{e}}-\frac{nm}{\varepsilon_{\mathrm{G}}}\rho_{\mathrm{e}}=\frac{\partial^{2}}{\partial t^{2}}\rho_{\mathrm{e}}+\omega_{\mathrm{p}}^{\prime 2}\rho_{\mathrm{e}}=0\ , \tag{94}$$

where the square of the modified plasma frequency $\omega_{\mathrm{p}}^{\prime}$ is given by

$$\omega_{\mathrm{p}}^{\prime 2}=\left(1-\frac{m^{2}}{q^{2}}\frac{Z_{\mathrm{G}}}{Z_{0}}\right)\frac{nq^{2}}{m\varepsilon_{0}}. \tag{95}$$

Here we have made use of the fact that $Z_0 = (c\varepsilon_0)^{-1}$ and that $Z_{\rm G} = (c\varepsilon_{\rm G})^{-1} = 4\pi G/c$. Comparing (95) with (84), we arrive at the following expression for Ξ:

$$\begin{aligned}\Xi &= 1 - \frac{m^2}{q^2}\frac{Z_{\rm G}}{Z_0} \\ &= 1 - \frac{4\pi\varepsilon_0 G m_{\rm e}^2}{e^2} \\ &\approx 1 - \frac{1}{4.2\times 10^{42}}\ .\end{aligned} \tag{96}$$

The fractional correction factor Ξ does indeed differ from unity by an extremely small amount, equal to the reciprocal of the ratio of the electrostatic force to the gravitational force between two electrons given by (1).

The implication of (96) for the *electro*dynamics of a superconductor is that the size of the modified plasma frequency given by (84) will be smaller than the standard value, albeit by a mere 4 parts in 10^{42}. Although this difference is extremely small, the fact that the modified plasma frequency is *smaller* rather than *larger* points to a surprising fact: the Cooper-pair holes created inside a superconducting film by an incident EM or GR microwave must be gravitationally repelled by, rather than attracted to, the corresponding Cooper pairs in the film, i.e., the holes must have the equivalent of negative mass and must therefore behave analogously to buoyant bubbles inside a fluid in the Earth's gravity. This would be a troubling result, were it not for the fact that the holes, like bubbles, cannot exist independently in the vacuum. The existence of negative-mass *pseudo*-particles (i.e., holes) within the film does *not* imply the possibility of shielding *static, longitudinal* gravito-electric fields, which requires the existence of *real* particles with negative mass in the vacuum. That is to say, the existence of these pseudo-particles does *not* imply the possibility of anti-gravity devices [30].

The real significance of Ξ lies in its impact on the *gravito*-electrodynamics of a superconducting film. In particular, the result given in (96) leads to an enhancement of the film's mass conductivity $\sigma_{\rm G}$ by the enormous factor of 4.2×10^{42}, which is what we have been calling the Heisenberg-Coulomb effect. Specifically, the expression for the mass conductivity given in (79) can now be reduced to

$$\sigma_{\rm G} \ = -i\frac{nq^2\Xi}{m\omega}\frac{Z_0}{Z_{\rm G}}\ , \tag{97}$$

or, equivalently [38],

$$\sigma_{1,{\rm G}} = 0 \quad \text{and} \quad \sigma_{2,{\rm G}} = -\frac{nq^2\Xi}{m\omega}\frac{Z_0}{Z_{\rm G}}\ . \tag{98}$$

Let us use this result to calculate the GR reflectivity of a superconducting film.

Recall the relationship given earlier in (23) between the inductance of the film and its nondissipative conductivity. Let us assume, once again, that the gravito-magnetic inductance is negligible

when compared to the gravitational kinetic inductance (which is justified in Appendix A). We can then equate the gravitational inductance of the film L_{G} with $L_{\mathrm{k,G}}$ and use (53), (56), (98) to express the latter as

$$\begin{aligned} L_{\mathrm{k,G}} &= \frac{1}{\omega\sigma_{2,\mathrm{G}}d} = -\frac{m}{nq^2\Xi}\frac{1}{d}\frac{Z_{\mathrm{G}}}{Z_0} \\ &\approx -\mu_{\mathrm{G}}d\left(\frac{\delta_{\mathrm{p,G}}}{d}\right)^2\frac{m^2Z_{\mathrm{G}}}{q^2Z_0} \\ &= -\mu_{\mathrm{G}}d\left(\frac{\delta_{\mathrm{p}}}{d}\right)^2 \\ &= -\mu_{\mathrm{G}}l'_{\mathrm{k,G}} \ , \end{aligned} \tag{99}$$

where the *corrected* gravitational kinetic inductance length scale $l'_{\mathrm{k,G}}$ is given by

$$l'_{\mathrm{k,G}} = d\left(\frac{\delta_{\mathrm{p}}}{d}\right)^2 \ . \tag{100}$$

But this is just the EM kinetic inductance length scale $l_{\mathrm{k,p}}$ that appears in the collisionless plasma model presented in Appendix B. Notice that this expression differs from the BCS expression given in (37) in Section 4 by a factor on the order of unity, i.e., d/ξ_0, which is due to the fact that the plasma model knows nothing of the BCS coherence length scale. Nonetheless, the appearance of δ_{p} in (100) highlights the importance of plasma concepts for correcting the approach adopted at the end of Section 6. The H-C effect reduces the GR kinetic inductance length scale $l_{\mathrm{k,G}}$ by 42 orders of magnitude, to the level of the EM kinetic inductance length scale $l_{\mathrm{k,p}}$ ($\approx l_{\mathrm{k}}$), thereby increasing the magnitude of the GR roll-off frequency $\omega_{\mathrm{r,G}}$ by the same factor, to the level of the EM roll-off frequency ω_{r}.

Two possible criticisms of this analysis immediately come to mind. First, the group velocity of a Cooper pair given by (69) is predicted to be superluminal, even for extremely small values of the dimensionless strain h_+ of an incident GR wave [33]. Using (71), (73), and (78) to solve for $|\mathbf{v}/c|$, one finds that

$$\left|\frac{\mathbf{v}}{c}\right| = \frac{1}{c}\frac{\Xi}{\Xi-1}|\mathbf{h}| = \frac{1}{2}\frac{\Xi}{\Xi-1}|h_+| \ . \tag{101}$$

Even for an arbitrarily chosen, extremely small value of $|h_+| \approx 10^{-40}$ (which, for a 6 GHz GR wave, corresponds to an incident power flux on the order of 10^{-16} W m^{-2}), the value given in (96) leads to a velocity roughly one hundred times the speed of light. This apparent violation of special relativity suggests that the response of a superconductor to a GR-wave field will in general be nonlinear, invalidating our assumption of linearity in (75).

However, group velocities much larger than c (infinite, even) have been experimentally demonstrated [39]. In particular, photon tunneling-time measurements confirm the "Wigner" transfer

time, which is a measure of an effective group velocity broadly applicable to quantum scattering processes. Wigner's analysis [40] assumes a *linear* relation between the initial and final states of a quantum system, and yields a transfer time that is proportional to the derivative of the phase of the system's transfer function with respect to the energy of the incident particle. In the present context, this implies that the Wigner time will be zero, since the phase of the Cooper-pair condensate remains constant everywhere, and stays unchanged with time and energy, due to first-order time-dependent perturbation theory (i.e., assuming that no pair-breaking or any other quantum excitation is allowed [15]). Returning to Figure 1, the Wigner time implies that an observer located at the center of mass of the superconductor who spots a Cooper pair at point B during the passage of the wave will see the pair disappear and then *instantaneously* re-appear at point A. This kind of *simultaneity* (as seen by the observer at the center of mass of the system) is a remarkable consequence of quantum theory, but it does not violate special relativity, nor does it invalidate the assumption of linearity.

We have already touched on the second criticism, namely, that the analysis presented here is defective because it does not register the BCS gap frequency. In particular, ohmic dissipation will occur at frequencies above the material's BCS gap frequency [2] and will damp out the free plasma oscillations that are otherwise predicted to occur in (95). In response, we note that these *dissipative* effects cannot alter the ratio given by (1) that appears in the *nondissipative* factor Ξ given in (96). Fundamentally, it is the strength of the Coulomb force, and not the strength of the gravitational force, that dictates the strength of a superconducting film's response to an incident GR wave.

VIII. THE NEGLIGIBILITY OF SINGLE-BOUNCE TRANSDUCTION

It is important to address the concern that an incoming GR wave will be partially or completely transduced into an outgoing EM wave by a superconducting film instead of being specularly reflected by the film. Recall that the Cooper pairs within the film *cannot* undergo free fall along with its lattice in the presence of an incident GR wave, contrary to a naive application of the EP to all particles. Instead, Cooper pairs must undergo *non-geodesic* motion, in contrast to the *geodesic* motion of the ions in the film's lattice. This leads to a non-zero quantum current density, one that carries mass and charge. Therefore, time-varying mass currents and time-varying charge currents will be generated by an incident GR wave. The latter will cause at least some of the incoming GR-wave energy to be transduced into an outgoing EM wave. More succinctly, the film will behave like an EM antenna. Appreciable transduction would be an interesting result in its own right, but it turns out to be negligible. The transduction effect is necessarily present in the interaction of a

superconducting film with a GR wave, but it does not undermine the film's ability to specularly reflect the wave.

The size of the transduction effect can be determined from a consideration of the charge supercurrent density generated within a superconducting film by an incident GR wave. Let us examine the case of a GR plane wave normally incident upon a superconducting film located at the plane $x = 0$, in the absence of any incident EM radiation. In this situation, the charge supercurrent generated by a GR wave will be generated as a *current sheet.* If a GR wave is incident upon the film only from the left, say, the charge supercurrent generated in the film will nonetheless radiate EM radiation symmetrically, i.e., in both the $+x$ and $-x$ directions. This follows from the bilateral symmetry of the current sheet, which takes the form

$$\mathbf{j}_{\mathrm{e}} = \mathbf{j}_0 \delta(x) \exp(-i\omega t) \tag{102}$$

around $x = 0$ (here and henceforth we suppress the polarization vectors of the currents and fields because they are all transverse to the x axis). The current sheet will radiate by coupling, via the Cooper pairs' charge $q = 2e$, to an electric field $\mathbf{E} = -\partial \mathbf{A}/\partial t$ (in the radiation gauge) and to a magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$.

Having chosen the radiation gauge, in which $\nabla \cdot \mathbf{A} = 0$ and in which the scalar potential is identically zero everywhere, we can begin with the EM wave equation in terms of $\mathbf{A}$ and $\mathbf{j}_{\mathrm{e}}$:

$$\nabla^2 \mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{j}_{\mathrm{e}} \ . \tag{103}$$

Let us assume once again that all time variations are sinusoidal at an angular frequency ω, so that we can make the replacements

$$\mathbf{A} \rightarrow \mathbf{A} \exp(-i\omega t) \text{ and } \mathbf{j}_{\mathrm{e}} \rightarrow \mathbf{j}_{\mathrm{e}} \exp(-i\omega t) \ . \tag{104}$$

Let us also take advantage of the symmetry inherent in the problem, so that we can reduce (103) to a Helmholtz equation in a single dimension for the transverse amplitudes A and j_{e}:

$$\frac{\partial^2 A}{\partial x^2} + k^2 A = -\mu_0 j_{\mathrm{e}} = -\mu_0 j_0 \delta(x) \ . \tag{105}$$

The delta function in (105) vanishes everywhere except at the origin $x = 0$, so that for all $x \neq 0$ this equation becomes a 1D homogeneous Helmholtz equation

$$\frac{\partial^2 A}{\partial x^2} + k^2 A = 0 \ . \tag{106}$$

By the principle of causality and the bilateral symmetry of the film, we can then restrict the possible solutions of this equation to *outgoing* plane waves symmetrically emitted from the film, so that

$$A = \alpha \exp(+ikx) \text{ for } x > 0 \tag{107a}$$

$$A = \alpha \exp(-ikx) \text{ for } x < 0 \tag{107b}$$

for the same value of α, which is determined by the strength of the delta function as follows:

$$\lim_{\varepsilon\to 0}\int_{-\varepsilon}^{+\varepsilon} dx\left(\frac{\partial^2 A}{\partial x^2}+k^2 A\right)=\lim_{\varepsilon\to 0}\left.\frac{\partial A}{\partial x}\right|_{-\varepsilon}^{+\varepsilon}$$

$$=-\mu_0 j_0\lim_{\varepsilon\to 0}\int_{-\varepsilon}^{+\varepsilon} dx\delta(x)=-\mu_0 j_0\ .$$

For $\varepsilon>0$, the derivatives of A are

$$\lim_{\varepsilon\to 0}\left.\frac{\partial A}{\partial x}\right|^{+\varepsilon}=\lim_{\varepsilon\to 0}\left(+ik\alpha\exp(+ik\varepsilon)\right)=+ik\alpha \tag{108}$$

and

$$\lim_{\varepsilon\to 0}\left.\frac{\partial A}{\partial x}\right|_{-\varepsilon}=\lim_{\varepsilon\to 0}\left(-ik\alpha\exp(-ik\varepsilon)\right)=-ik\alpha\ . \tag{109}$$

Hence

$$\lim_{\varepsilon\to 0}\left.\frac{\partial A}{\partial x}\right|_{-\varepsilon}^{+\varepsilon}=+2ik\alpha\ . \tag{110}$$

Therefore, the amplitude α of the radiation field A emitted from the charge current sheet of strength j_0 generated by an incident GR wave is given by

$$\alpha=i\frac{1}{2}\frac{\mu_0 j_0}{k}\ . \tag{111}$$

For a very thin film of thickness d, the delta function $\delta(x)$ is approximately

$$\delta(x)\approx\frac{1}{d} \tag{112}$$

inside the film and zero outside, since then

$$\int_{-d/2}^{d/2}\delta(x)dx=1\ , \tag{113}$$

which implies that

$$\alpha=i\frac{1}{2}\frac{Z_0 d}{\omega}j_{\mathrm{e}}\ . \tag{114}$$

As we saw in the previous section, an incident GR wave generates within a superconducting film not only an E_{G} field but an internal E field as well. In each case, the tangential component of the field must be continuous across the superconductor-vacuum interface. Since there is no incoming E field, this continuity condition requires the appearance of an outgoing E field, which is to say that the charge supercurrent generated by the GR wave will cause the film to behave like an antenna and

radiate EM waves. For the same sinusoidal time dependence $\exp(-i\omega t)$ of all fields and currents, and ignoring spatial dependence, we know that

$$E = -\frac{\partial}{\partial t} A = i\omega A = i\omega\alpha \; . \tag{115}$$

Inserting (114) into this expression, we see that the relationship between the charge supercurrent j_{e} in the current sheet and the E field both outside and inside the film will be given by

$$j_{\mathrm{e}} = nqv = -\frac{2}{Z_0 d} E \; . \tag{116}$$

The charge conductivity of the film stemming from its behavior as an EM antenna in the presence of a GR wave is thus given by

$$\sigma_{\mathrm{e}} = -\frac{2}{Z_0 d} \; . \tag{117}$$

Now, it must be possible to re-express this charge conductivity as the real part of the complex mass conductivity. The justification for this step is that the EM radiation produced in transduction from the incident GR wave leads to power loss from the wave that escapes to infinity, never to return. Hence the transduction effect is a lossy process in the GR wave sector, which is no different from any other irreversible, ohmic process, and can therefore be characterized as the real part of the mass conductivity. Multiplying each side of (116) by m/q and using the relationship between E and E_{G} given earlier in (77), one finds that the lossy component of the mass current density $\mathbf{j}_{\mathrm{G}}$ arising from the transduction of the incident GR wave into an EM wave is given by

$$j_{\mathrm{loss,G}} = -\frac{2}{Z_0 d} \frac{m^2}{q^2} \frac{1}{\Xi - 1} E_{\mathrm{G}} \; . \tag{118}$$

The real part of the mass conductivity $\sigma_{1,\mathrm{G}}$ of the film due to the dissipative loss by transduction into the escaping EM radiation is therefore given by

$$\sigma_{1,\mathrm{G}} = -\frac{2}{Z_0 d} \frac{m^2}{q^2} \frac{1}{\Xi - 1} = \frac{2}{Z_{\mathrm{G}} d} \; , \tag{119}$$

where we have taken advantage of the fact that

$$\Xi - 1 = \left(1 - \frac{m^2 Z_{\mathrm{G}}}{q^2 Z_0} \right) - 1 = -\frac{m^2 Z_{\mathrm{G}}}{q^2 Z_0} \; . \tag{120}$$

We can now use (119) in conjunction with the nondissipative conductivity $\sigma_{2,\mathrm{G}}$ given in (98)

$$\sigma_{2,\mathrm{G}} = -\Xi \frac{nq^2}{m\omega} \frac{Z_0}{Z_{\mathrm{G}}} \tag{121}$$

to determine whether loss into EM radiation will undermine the possibility of GR-wave reflection. We begin by recalling that the full version of the GR-reflection formula is given by

$$\mathcal{R}_{\mathrm{G}} = \left\{ \left(1 + 2\frac{\sigma_{1,\mathrm{G}}}{\sigma_{1,\mathrm{G}}^2 + \sigma_{2,\mathrm{G}}^2} \frac{1}{Z_{\mathrm{G}} d} \right)^2 + \left(2\frac{\sigma_{2,\mathrm{G}}}{\sigma_{1,\mathrm{G}}^2 + \sigma_{2,\mathrm{G}}^2} \frac{1}{Z_{\mathrm{G}} d} \right)^2 \right\}^{-1} . \tag{122}$$

Now let us define the parameter Σ, which is the dimensionless ratio of the squares of the two mass conductivities given in (119) and (121)

$$\Sigma \equiv \left(\frac{\sigma_{1,\mathrm{G}}}{\sigma_{2,\mathrm{G}}} \right)^2 = \left(\frac{2m\omega}{\Xi n q^2 Z_0 d} \right)^2 = \left(\frac{2}{\pi} \frac{\omega_{\mathrm{d}}}{\omega_{\mathrm{p}}^{\prime 2}} \omega \right)^2 , \tag{123}$$

where $\omega_{\mathrm{p}}^{\prime}$ is the modified plasma frequency (84) and $\omega_{\mathrm{d}} = \pi c/d$ is a characteristic frequency associated with the thickness of the film d (i.e., the resonance frequency for its lowest standing-wave mode). In general, it will be the case that Σ is much less than unity when the frequency of the incident wave is

$$\omega \ll \frac{\pi}{2} \frac{\omega_{\mathrm{p}}^{\prime 2}}{\omega_{\mathrm{d}}} = 1.1 \times 10^{16} \text{ rad s}^{-1}. \tag{124}$$

The microwave frequencies of interest fall well below this limit, so we can simplify (122) to

$$\mathcal{R}_{\mathrm{G}} = \left\{ \left(1 + 2\frac{\sigma_{1,\mathrm{G}}}{\sigma_{2,\mathrm{G}}^2} \frac{1}{Z_{\mathrm{G}} d} \right)^2 + \left(2\frac{1}{\sigma_{2,\mathrm{G}}} \frac{1}{Z_{\mathrm{G}} d} \right)^2 \right\}^{-1} . \tag{125}$$

We can then substitute (119) and (121) into (125) to obtain

$$\mathcal{R}_{\mathrm{G}} = \left\{ (1+\Sigma)^2 + \Sigma \right\}^{-1} . \tag{126}$$

For $\omega = 2\pi \times (6 \text{ GHz})$, we see from (123) that

$$\Sigma = 1.3 \times 10^{-11} \tag{127}$$

and thus that

$$\mathcal{R}_{\mathrm{G}} \approx (1 + 3\Sigma)^{-1} = \left(1 + 3.8 \times 10^{-11}\right)^{-1} , \tag{128}$$

which implies a reflectivity very close to unity. Thus the dissipation (i.e., transduction) of an incident GR wave in the form of outgoing EM radiation will not interfere with the film's ability to specularly reflect GR waves.

As a check on this conclusion, let us examine the ratio η of the power *lost* in the form of outgoing EM radiation to the power *reflected* in the form of outgoing GR radiation, using the reasonable assumption that the film acts as a current source in both sectors. Thus,

$$\eta = \frac{\langle \mathcal{P}_{\rm EM} \rangle}{\langle \mathcal{P}_{\rm GR} \rangle} = \frac{\left\langle I_{\rm e}^2 \right\rangle Z_0}{\left\langle I_{\rm G}^2 \right\rangle Z_{\rm G}} = \frac{\left\langle I_{\rm loss,G}^2 \right\rangle Z_{\rm G}}{\left\langle I_{\rm G}^2 \right\rangle Z_{\rm G}} \tag{129}$$
$$= \frac{\left\langle I_{\rm loss,G}^2 \right\rangle}{\left\langle I_{\rm G}^2 \right\rangle} = \frac{\left\langle j_{\rm loss,G}^2 \right\rangle}{\left\langle j_{\rm G}^2 \right\rangle} = \frac{\sigma_{1,\rm G}^2}{\sigma_{2,\rm G}^2} = \Sigma \ .$$

The value for Σ given in (127) implies that a negligible fraction of the power of the incoming GR microwave will be lost through transduction into an outgoing EM wave. A superconducting film at temperatures sufficiently near $T = 0$ will indeed be a highly reflective mirror for GR microwaves but a highly inefficient transducer of GR microwaves into EM microwaves.

In the parallel case of EM-wave reflection, we can once again take into account the possibility of transduction by introducing a real term into the EM conductivity that corresponds to loss into the GR sector (i.e., into an outgoing GR wave). The resulting real and imaginary parts of the complex charge conductivity of the film can then be shown to be

$$\sigma_1 = \frac{2}{Z_0 d} \quad \text{and} \quad \sigma_2 = \Xi \frac{nq^2}{m\omega} \ , \tag{130}$$

where σ_1 is the dissipative part of the complex charge conductivity corresponding to loss by transduction into outgoing GR radiation. The film will radiate as a GR antenna because of the appearance of a quadrupolar pattern of *mass* supercurrents (when driven by a TEM_{11} incident EM plane-wave mode) that couples, via the Cooper pairs' mass $m = 2m_{\rm e}$, to a gravito-electric field $\mathbf{E}_{\rm G} = -\partial \mathbf{h}/\partial t$ (in the radiation gauge) and to a gravito-magnetic field $\mathbf{B}_{\rm G} = \nabla \times \mathbf{h}$, where $\mathbf{h}$ is the gravitational analog of the electromagnetic vector potential $\mathbf{A}$.

In fact, (130) leads to an expression for Σ identical to the one given in (123). The subsequent analysis then proceeds unaltered, confirming that the model developed here is also consistent with the prediction that at temperatures sufficiently near $T = 0$ a superconducting film will be a highly reflective mirror for EM microwaves but a highly inefficient transducer of EM microwaves into GR microwaves. Importantly, this prediction is consistent with the experimental results of Glover and Tinkham [1].

IX. CONSERVATION OF ENERGY IN THE REFLECTION PROCESS

Having shown that a superconducting film can specularly reflect a GR wave and that transduction will not substantially impede this behavior, we turn finally to the question of whether the

expressions for $\sigma_{1,\mathrm{G}}$ and $\sigma_{2,\mathrm{G}}$ given above in (119) and (121) are consistent with the conservation of energy. This basic physical principle requires that the absorptivity, reflectivity, and transmissivity of the film sum to unity:

$$\begin{aligned} \mathcal{A}_\mathrm{G} + \mathcal{R}_\mathrm{G} + \mathcal{T}_\mathrm{G} &= 1 \\ = \mathcal{A}_\mathrm{G}\mathcal{R}_\mathrm{G}\mathcal{T}_\mathrm{G} \times \left(\frac{1}{\mathcal{A}_\mathrm{G}\mathcal{R}_\mathrm{G}} + \frac{1}{\mathcal{A}_\mathrm{G}\mathcal{T}_\mathrm{G}} + \frac{1}{\mathcal{R}_\mathrm{G}\mathcal{T}_\mathrm{G}} \right) \ . \end{aligned} \tag{131}$$

From the analysis presented in Sections 3-6, we know that the reciprocal of the GR transmissivity is given by

$$\frac{1}{\mathcal{T}_\mathrm{G}} = \left(1 + \frac{1}{2}\sigma_{1,\mathrm{G}} Z_\mathrm{G} d\right)^2 + \left(\frac{1}{2}\sigma_{2,\mathrm{G}} Z_\mathrm{G} d\right)^2 \tag{132}$$

and that the reciprocal of the GR reflectivity is given by

$$\begin{aligned} \frac{1}{\mathcal{R}_\mathrm{G}} = &\left(1 + 2\frac{\sigma_{1,\mathrm{G}}}{\sigma_{1,\mathrm{G}}^2 + \sigma_{2,\mathrm{G}}^2} \frac{1}{Z_\mathrm{G} d}\right)^2 \\ &+ \left(2\frac{\sigma_{2,\mathrm{G}}}{\sigma_{1,\mathrm{G}}^2 + \sigma_{2,\mathrm{G}}^2} \frac{1}{Z_\mathrm{G} d}\right)^2 \ . \end{aligned} \tag{133}$$

We can determine the reciprocal of the GR absorptivity $\mathcal{A}_\mathrm{G}$ by considering the work done by a gravito-electric field $\mathbf{E}_\mathrm{G}$ to move a mass m by an infinitesimal displacement $d\mathbf{x}$:

$$dW = \mathbf{F} \cdot d\mathbf{x} = m\mathbf{E}_\mathrm{G} \cdot d\mathbf{x} \ . \tag{134}$$

The rate of work being done, i.e., the instantaneous power $\mathtt{P}$ delivered by the field to the mass, is

$$\mathtt{P} = \mathbf{F} \cdot \frac{d\mathbf{x}}{dt} = m\mathbf{E}_\mathrm{G} \cdot \frac{d\mathbf{x}}{dt} \ . \tag{135}$$

Let n be the number density of mass carriers moving with velocity

$$\mathbf{v} = \frac{d\mathbf{x}}{dt} \ , \tag{136}$$

so that the mass current density $\mathbf{j}_\mathrm{G}$ is

$$\mathbf{j}_\mathrm{G} = nm\mathbf{v} \ . \tag{137}$$

Then the instantaneous power delivered by the field to the mass carriers per unit volume moving in a small volume V is

$$\mathcal{P} = \frac{\mathtt{P}}{V} = nm\mathbf{E}_\mathrm{G} \cdot \frac{d\mathbf{x}}{dt} = \mathbf{j}_\mathrm{G} \cdot \mathbf{E}_\mathrm{G} \ , \tag{138}$$

where $\mathbf{j}_\mathrm{G}$ and $\mathbf{E}_\mathrm{G}$ are real quantities. Let us, however, generalize this expression and represent the current and field by the complex quantities

$$\mathbf{j}_\mathrm{G}{=}\mathbf{j}_{0,\mathrm{G}}\exp(-i\omega t) \text{ and} \tag{139a}$$

$$\mathbf{E}_\mathrm{G}{=}\mathbf{E}_{0,\mathrm{G}}\exp(-i\omega t) \ . \tag{139b}$$

Then

$$\mathrm{Re}\,\mathbf{j}_\mathrm{G}{=}\frac{1}{2}\left[\mathbf{j}_{0,\mathrm{G}}\exp(-i\omega t)+\mathbf{j}^*_{0,\mathrm{G}}\exp(i\omega t)\right] \tag{140}$$

and

$$\mathrm{Re}\,\mathbf{E}_\mathrm{G}{=}\frac{1}{2}\left[\mathbf{E}_{0,\mathrm{G}}\exp(-i\omega t)+\mathbf{E}^*_{0,\mathrm{G}}\exp(i\omega t)\right] \ . \tag{141}$$

The *real* instantaneous power per unit volume expressed in terms of this *complex* current and field is given by

$$\begin{aligned}\mathcal{P} &= \mathrm{Re}\,\mathbf{j}_\mathrm{G}\cdot\mathrm{Re}\,\mathbf{E}_\mathrm{G} \\ &{=}\frac{1}{2}\left[\mathbf{j}_{0,\mathrm{G}}\exp(-i\omega t)+\mathbf{j}^*_{0,\mathrm{G}}\exp(i\omega t)\right]\cdot \\ &\quad\frac{1}{2}\left[\mathbf{E}_{0,\mathrm{G}}\exp(-i\omega t)+\mathbf{E}^*_{0,\mathrm{G}}\exp(i\omega t)\right].\end{aligned} \tag{142}$$

But the time average over one wave-period $T=2\pi/\omega$ of each second harmonic term in this expression vanishes because

$$\frac{1}{T}\int_0^T dt\left[\mathbf{j}_{0,\mathrm{G}}\cdot\mathbf{E}_{0,\mathrm{G}}\exp(-2i\omega t)\right]=0 \tag{143a}$$

$$\frac{1}{T}\int_0^T dt\left[\mathbf{j}^*_{0,\mathrm{G}}\cdot\mathbf{E}^*_{0,\mathrm{G}}\exp(+2i\omega t)\right]=0 \ , \tag{143b}$$

leaving only the DC cross terms

$$\langle\mathcal{P}\rangle=\frac{1}{4}\left(\mathbf{j}_{0,\mathrm{G}}\cdot\mathbf{E}^*_{0,\mathrm{G}}+\mathbf{j}^*_{0,\mathrm{G}}\cdot\mathbf{E}_{0,\mathrm{G}}\right) \ , \tag{144}$$

which can be re-expressed as

$$\langle\mathcal{P}\rangle=\frac{1}{2}\,\mathrm{Re}(\mathbf{j}^*_\mathrm{G}\cdot\mathbf{E}_\mathrm{G}) \ . \tag{145}$$

Let us apply this result for the time-averaged power density to a superconducting film by recalling that the relevant gravito-electric field is the field *inside* the film, so that

$$\langle\mathcal{P}\rangle=\frac{1}{2}\,\mathrm{Re}(\mathbf{j}^*_\mathrm{G}\cdot\mathbf{E}_{\text{G-inside}}), \tag{146}$$

where the gravitational analog of Ohm's law is

$$\mathbf{j}_{\mathrm{G}} = \sigma_{\mathrm{G}} \mathbf{E}_{\text{G-inside}} \ . \tag{147}$$

Therefore,

$$\begin{aligned} \langle \mathcal{P} \rangle &= \frac{1}{2} \left(\mathrm{Re}\, \sigma_{\mathrm{G}}^* \right) \left(\mathbf{E}_{\text{G-inside}}^* \cdot \mathbf{E}_{\text{G-inside}} \right) \\ &= \frac{1}{2} \mathrm{Re} \left(\sigma_{1,\mathrm{G}} - i \sigma_{2,\mathrm{G}} \right) \left| \mathbf{E}_{\text{G-inside}} \right|^2 \\ &= \frac{1}{2} \sigma_{1,\mathrm{G}} \left| \mathbf{E}_{\text{G-inside}} \right|^2 \ . \end{aligned} \tag{148}$$

As in the electromagnetic case discussed in Section 3, the gravito-electric field inside the film will be related to the incident gravito-electric field as follows:

$$\begin{aligned} \mathbf{E}_{\text{G-inside}} &= (1 - r_{\mathrm{G}}) \mathbf{E}_{\text{G-incident}} \\ &= t_{\mathrm{G}} \mathbf{E}_{\text{G-incident}} \\ &= \mathbf{E}_{\text{G-transmitted}} \ , \end{aligned} \tag{149}$$

where r_{G} is the amplitude reflection coefficient and t_{G} is the amplitude transmission coefficient. Thus the time-averaged power dissipated inside the entire volume Ad of the film, where A is its area (an arbitrarily large quantity) and d is its thickness, is given by

$$\begin{aligned} \langle \mathcal{P} \rangle \, Ad &= \frac{1}{2} \sigma_{1,\mathrm{G}} t_{\mathrm{G}}^* t_{\mathrm{G}} \left| \mathbf{E}_{\text{G-incident}} \right|^2 Ad \\ &= \frac{A \sigma_{1,\mathrm{G}} d}{2} \mathcal{T}_{\mathrm{G}} \left| \mathbf{E}_{\text{G-incident}} \right|^2 \ , \end{aligned} \tag{150}$$

where $\mathcal{T}_{\mathrm{G}} = t_{\mathrm{G}}^* t_{\mathrm{G}}$ is the transmittivity of the film.

The magnitude of the time-averaged Poynting vector of the incident wave traveling in the direction $\hat{\mathbf{k}}$ is given by an expression similar to (145), viz.,

$$\begin{aligned} \langle \mathcal{S} \rangle &= \frac{1}{2} \hat{\mathbf{k}} \cdot \mathrm{Re} \left(\mathbf{E}_{\text{G-incident}}^* \times \mathbf{H}_{\text{G-incident}} \right) \\ &= \frac{1}{2} \frac{1}{Z_{\mathrm{G}}} \mathrm{Re} (\mathbf{E}_{\text{G-incident}}^* \cdot \mathbf{E}_{\text{G-incident}}) \\ &= \frac{1}{2 Z_{\mathrm{G}}} \left| \mathbf{E}_{\text{G-incident}} \right|^2 \ , \end{aligned} \tag{151}$$

from which it follows that the power incident on the area A of the film is

$$\langle S \rangle \, A = \frac{1}{2 Z_{\mathrm{G}}} \left| \mathbf{E}_{\text{G-incident}} \right|^2 A \ . \tag{152}$$

Thus the absorptivity $\mathcal{A}_{\mathrm{G}}$, which is the ratio of the time-averaged power dissipated inside the film to the time-averaged power incident on the film, is given by

$$\mathcal{A}_{\mathrm{G}} = \frac{\langle \mathcal{P} \rangle \, Ad}{\langle S \rangle \, A} = \mathcal{T}_{\mathrm{G}} \sigma_{1,\mathrm{G}} Z_{\mathrm{G}} d \ . \tag{153}$$

Inserting the reciprocal of (132) for $\mathcal{T}_{\mathrm{G}}$ into (153) and taking the reciprocal of the new expression, we find that

$$\frac{1}{\mathcal{A}_{\mathrm{G}}} = \frac{(1+\frac{1}{2}\sigma_{1,\mathrm{G}} Z_{\mathrm{G}} d)^2 + (\frac{1}{2}\sigma_{2,\mathrm{G}} Z_{\mathrm{G}} d)^2}{\sigma_{1,\mathrm{G}} Z_{\mathrm{G}} d} \ . \tag{154}$$

A calculation confirms that the expressions given in (132), (133), and (154) are in fact consistent with the requirement that the absorptivity, reflectivity, and transmissivity sum to unity. When we insert (132), (133), and (154) into the right-hand side of (131), we obtain a single equation with two variables, $\sigma_{1,\mathrm{G}}$ and $\sigma_{2,\mathrm{G}}$. Thus the conservation of energy in the case of the interaction between a GR wave and a superconducting film depends solely on the relation between $\sigma_{1,\mathrm{G}}$ and $\sigma_{2,\mathrm{G}}$.

Recalling the parameter Σ introduced in (123), which characterizes the relation between $\sigma_{1,\mathrm{G}}$ and $\sigma_{2,\mathrm{G}}$, we can re-express the reciprocals of $\mathcal{A}_{\mathrm{G}}$, $\mathcal{R}_{\mathrm{G}}$, and $\mathcal{T}_{\mathrm{G}}$ as

$$\frac{1}{\mathcal{A}_{\mathrm{G}}} = 2 + \frac{1}{2\Sigma} \tag{155a}$$

$$\frac{1}{\mathcal{R}_{\mathrm{G}}} = \frac{2\Sigma^2 + 10\Sigma^3 + 8\Sigma^4}{2\Sigma^2 + 4\Sigma^3 + 2\Sigma^4} \tag{155b}$$

$$\frac{1}{\mathcal{T}_{\mathrm{G}}} = 4 + \frac{1}{\Sigma} \ . \tag{155c}$$

One then finds that

$$\mathcal{A}_{\mathrm{G}}\mathcal{R}_{\mathrm{G}}\mathcal{T}_{\mathrm{G}} = \frac{2\Sigma^2 + 4\Sigma^3 + 2\Sigma^4}{1 + 13\Sigma + 60\Sigma^2 + 112\Sigma^3 + 64\Sigma^4} \tag{156}$$

and that

$$\left(\frac{1}{\mathcal{A}_{\mathrm{G}}\mathcal{R}_{\mathrm{G}}} + \frac{1}{\mathcal{A}_{\mathrm{G}}\mathcal{T}_{\mathrm{G}}} + \frac{1}{\mathcal{R}_{\mathrm{G}}\mathcal{T}_{\mathrm{G}}}\right) = \frac{1 + 13\Sigma + 60\Sigma^2 + 112\Sigma^3 + 64\Sigma^4}{2\Sigma^2 + 4\Sigma^3 + 2\Sigma^4} \ . \tag{157}$$

But these two expressions are just the reciprocals of one another, confirming (131) in the GR case.

In the EM case, the corresponding real and imaginary parts of the complex *charge* conductivity are given by

$$\sigma_1 = \frac{2}{Z_0 d} \tag{158}$$

$$\sigma_2 = \Xi \frac{nq^2}{m\omega} \ . \tag{159}$$

The fact that the ratio of the squares of these two conductivities is once again

$$\left(\frac{\sigma_1}{\sigma_2}\right)^2 = \frac{4m^2\omega^2}{\Xi^2 n^2 q^4 Z_0^2 d^2} = \Sigma \tag{160}$$

is a strong hint that (131) will be similarly satisfied in the EM case. In fact, the expressions given above for $1/\mathcal{A}_{\mathrm{G}}$, $1/\mathcal{R}_{\mathrm{G}}$, and $1/\mathcal{T}_{\mathrm{G}}$ in (155) carry over without alteration into the EM case, so that

the subsequent steps of the derivation proceed exactly as above. Consequently, we can say that the formalism presented here obeys energy conservation both in the case of GR reflection with EM loss and in the case of EM reflection with GR loss. This is a strong self-consistency check of the entire calculation.

X. EXPERIMENTAL AND THEORETICAL IMPLICATIONS

Most of the experiments presently being conducted on gravitational radiation aim to passively detect GR waves originating from astrophysical sources. The specular reflection of GR waves at microwave frequencies from superconducting thin films due to the Heisenberg-Coulomb effect would allow for a variety of new experiments, all of which could be performed in a laboratory setting and some of which would involve mesoscopic quantum objects. Here we identify several such experiments that should be technologically feasible, commenting briefly on their interrelations and broader theoretical implications.

Consider first a conceptually simple test of the physics behind the Heisenberg-Coulomb effect itself. In this experiment, two horizontally well-separated, noninteracting superconducting bodies are allowed to fall freely in the non-uniform gravitational field of the Earth. The tidal forces acting on the two bodies, which are like the tidal forces caused by a low-frequency GR wave, cause them to converge as they fall freely toward the center of the earth. Although the gap-protected, global quantum mechanical phase of the Cooper pairs forces each pair to remain motionless with respect to the center of mass of its own body, this does nothing to prevent the two bodies from converging during free fall. The trajectories of these two superconducting bodies – recall that they have decohered due to interactions with their environment and are therefore spatially well separated – must be identical to those of *any* two noninteracting, freely falling massive bodies, in accord with the EP.

Now connect the two bodies by a thin, slack, arbitrarily long, superconducting wire, so that they become a single, simply-connected, coherent superconducting system. From a mechanical point of view, the negligible Hooke's constant of the wire allows each body to move freely, one relative to the other. In this case, the characteristic frequency of the interaction between the bodies and the gravitational field, which is given by the inverse of the free-fall time, is far below the BCS gap frequency and far above the simple harmonic resonance frequency of the two-bodies-plus-wire system. According to first-order time-dependent perturbation theory, then, the nonlocalizable Cooper pairs of the two coherently connected bodies must remain motionless with respect to the center of mass of the *entire* system, as seen by a distant inertial observer. This follows, as we

have argued in Section 2, from the gap-protected, global, quantum mechanical phase of the Cooper pairs, which is at root a consequence of the UP. On the other hand, the ions of the two coherently connected bodies will attempt to converge toward each other during free fall, since they want to follow geodesics in accord with the EP.

In this experimental "tug-of-war" between the Uncertainty Principle and the Equivalence Principle, which principle prevails? When the temperature is low enough to justify ignoring the effect of any residual normal electrons (i.e., when the temperature is less than roughly half the critical temperature, so that the BCS gap is sufficiently close to its value at absolute zero [2]), we believe the EP will be completely overcome by the UP. This must be the case because the charge separation that would otherwise result as the ions converged while the Cooper pairs remained motionless (with respect to a distant inertial observer) would generate an unfavorable, higher-energy configuration of the system. The *quantum mechanical* Cooper pairs must drag the *classical* ionic lattice into co-motion with them, so that the coherently connected bodies *depart* from geodesic motion. That is to say, the bodies must maintain a *constant distance* from one another as they fall. If two coherently connected superconducting bodies *were* to converge like any two noninteracting bodies, one would have to conclude that the UP had failed with respect to the EP, i.e., that the EP is more universal and fundamental in its application to all objects than the UP. We do not believe this to be the case.

Theories that propose an "intrinsic collapse of the wavefunction" or "objective state reduction," through some decoherence mechanism, whether by means of a stochastic process that leverages the entanglement of object and environment (as originally proposed by Ghirardi, Rimini, and Weber [41]), or by means of a sufficiently large change in the gravitational self-energy associated with different mass configurations of a system (as proposed by Penrose [42]), would imply the failure of the Superposition Principle, and thus of the Uncertainty Principle, in the experiment outlined above. The existence of any such mechanism would destroy the Heisenberg-Coulomb effect, but it would also pose a serious problem for any quantum theory of gravity.

A straightforward geometrical calculation for the free-fall experiment outlined above shows that the convergence of two noninteracting massive bodies initially separated by several centimeters would be on the order of microns for free-fall distances presently attainable in aircraft-based zero-gravity experiments. Though small, this degree of convergence is readily measureable by means of laser interferometry. The exact *decrease*, if any, in the convergence measured for two coherently connected superconducting bodies, relative to the decrease measured for the same two bodies when the coherent connection is broken, would allow one to measure the strength of the Heisenberg-Coulomb effect, with null convergence corresponding to maximal deflection from free fall.

The specular reflection of GR waves from superconducting films, which we have argued follows from the Heisenberg-Coulomb effect (see Section 7), might also allow for the detection of a gravitational Casimir-like force (we thank Dirk Bouwmeester for this important suggestion). In the EM case, an attractive force between two nearby metallic plates is created by radiation pressure due to quantum fluctuations in the EM vacuum energy. If the two plates were made of a type I superconducting material, it should be possible to detect a change in the attractive force between them, due to the additional coupling of the plates to quantum fluctuations in the GR vacuum energy, as the plates were lowered through their superconducting transition temperature. Observation of the gravitational analog of the Casimir force could be interpreted as evidence for the existence of quantum fluctuations in gravitational fields, and hence as evidence for the need to quantize gravity. If no analog of the Casimir force were observed despite confirmation of the Heisenberg-Coulomb effect in free-fall experiments, one would be forced to conclude either that gravitational fields are not quantizable or that something other than the Heisenberg-Coulomb effect is wrong with our "mirrors" argument.

In Sections 8 and 9, we discussed the transduction of GR waves to EM waves and vice-versa. Although we showed that transduction in either direction will be highly inefficient in the case of a single superconducting film, experimentally significant efficiencies in both directions may be attainable in the case of *a pair of charged superconductors* [43]. This would lead to a number of experimental possibilities, all of which employ the same basic apparatus: two levitated (or suspended) and electrically charged superconducting bodies that repel one another electrostatically even as they attract one another gravitationally. For small bodies, it is experimentally feasible to charge the bodies to "criticality," i.e., to the point at which the forces of repulsion and attraction cancel [43]. At criticality, the apparatus should become an effective transducer of incoming GR radiation, i.e., it should enable 50% GR-to-EM transduction efficiency. By time-reversal symmetry, it should also become an effective transducer of incoming EM radiation, i.e., it should also enable 50% EM-to-GR transduction efficiency. Chiao has previously labeled this type of apparatus a "quantum transducer" [43].

Two variations on a single-transducer experiment could provide new and compelling, though still indirect, evidence for the existence of GR waves. First, an electromagnetically isolated transducer should generate an EM signal in the presence of an incoming GR wave, since the transducer should convert half the power contained in any incoming GR wave into a detectable outgoing EM wave. This might allow for the detection of the cosmic gravitational-wave background (CGB) at microwave frequencies, assuming that certain cosmological models of the extremely early Big Bang are correct [44]. If no transduced EM signal were detected despite confirmation of the H-C effect, one would be

forced to conclude either that something is wrong with the “mirrors” argument or the GR-to-EM “transduction” argument, or that there is no appreciable CGB at the frequency of investigation.

A single quantum transducer should also behave anomalously below its superconducting transition temperature in the presence of an incoming EM wave (we thank Ken Tatebe for this important suggestion). By the principle of the conservation of energy, an EM receiver directed at the transducer should register a significant *drop* in reflected power when the transducer is “turned on” by lowering its temperature below the transition temperature of the material, since energy would then be escaping from the system in the form of invisible (transduced) GR waves. If no drop in reflected power were observed despite confirmation of GR-to-EM transduction in the experiment outlined in the previous paragraph, one would need to reconsider the validity of the principle of time-reversal symmetry in the argument for EM-to-GR transduction.

Finally, if an efficient quantum transducer were to prove experimentally feasible, two transducers operating in tandem would open up the possibility of GR-wave communication. As a start, a gravitational Hertz-like experiment should be possible. An initial transducer could be used to partially convert an incoming EM into an outgoing GR wave. A second transducer, spatially separated and electromagnetically isolated from the first, could then be used to partially back-convert the GR wave generated by the first transducer into a detectable EM wave. The same two-transducer arrangement could also be used to confirm the predicted speed and polarization of GR waves. Of course, wireless communication via GR waves would be highly desirable, since all normal matter is effectively transparent to GR radiation. Such technology would also open up the possibility of wireless power transfer over long distances. On the other hand, if a Hertz-like arrangement were to yield a null result despite the success of the previously outlined single-transducer experiments, one would infer that the success of those experiments was due to something other than the existence of GR waves.

In summary, a new class of laboratory-scale experiments at the interface of quantum mechanics and gravity follows if the argument presented here for superconducting GR-wave mirrors is correct. Such experiments could be a boon to fundamental physics. For example, one could infer from the experimental confirmation of a gravitational Casimir effect that gravitational fields are in fact quantized. Confirmation of the Heisenberg-Coulomb effect would also point to the need for a unified *gravito-electrodynamical* theory for weak, but quantized, gravitational and electromagnetic fields interacting with nonrelativistic quantum mechanical matter. Such a theory would fall far short of the ultimate goal of unifying all known forces of nature into a “theory of everything,” but it would nonetheless be a very useful theory to have.

APPENDIX A: THE MAGNETIC AND KINETIC INDUCTANCES OF A THIN METALLIC FILM

The EM inductance L of the superconducting film is composed of two parts: the magnetic inductance L_{m}, which arises from the magnetic fields established by the charge supercurrents, carried by the Cooper pairs, and the kinetic inductance L_{k}, which arises from the Cooper pairs' inertial mass [8]. Using (37) and the values of $\xi_0 = 83$ nm and $\delta_{\mathrm{p}} = \lambda_{\mathrm{L}} = 37$ nm for Pb at microwave frequencies, one finds for our superconducting film that l_{k} is on the order of 10^{-5} m and that L_{k} is on the order of 10^{-11} henries.

L_{m} can be found using the magnetic potential energy relations

$$U = \int \frac{B^2}{2\mu_0}\, d^3x = \frac{1}{2} L_{\mathrm{m}} I^2 \ , \tag{A1}$$

where U is the magnetic potential energy, B is the magnetic induction field, and I is the (uniform) current flowing through the film. Thus,

$$L_{\mathrm{m}} = \int \frac{B^2}{I^2 \mu_0} d^3x \ . \tag{A2}$$

A closed-form, symbolic expression for this integral is complicated for the geometry of a film, but numerical integration shows that in the case of a Pb film with dimensions 1 cm $\times$ 1 cm $\times$ 2 nm, L_{m} is on the order of at most 10^{-15} henries, which is much smaller than L_{k}. The experiments of Glover and Tinkham [1] corroborate the validity of this approximation. Thus, we can safely neglect the magnetic inductance L_{m} in our consideration of L.

A comparison of this result for L_{m} with the result for $L_{\mathrm{m,G}}$ in the gravitational sector reveals that

$$\frac{L_{\mathrm{m,G}}}{L_{\mathrm{m}}} = \frac{\mu_{\mathrm{G}}}{\mu_0} \ . \tag{A3}$$

Recall now that the expression for $l'_{\mathrm{k,G}}$ given by (100) is

$$l'_{\mathrm{k,G}} = d \left(\frac{\delta_{\mathrm{p}}}{d} \right)^2 \ , \tag{A4}$$

which is just the expression for $l_{\mathrm{k,p}}$ ($\approx l_{\mathrm{k}}$) derived in Appendix B below. Thus we see that

$$\frac{L_{\mathrm{k,G}}}{L_{\mathrm{k}}} = \frac{\mu_{\mathrm{G}} l'_{\mathrm{k,G}}}{\mu_0 l_{\mathrm{k}}} \approx \frac{\mu_{\mathrm{G}}}{\mu_0} \ . \tag{A5}$$

From (A3) and (A5), it follows that

$$\frac{L_{\mathrm{m,G}}}{L_{\mathrm{k,G}}} \approx \frac{L_{\mathrm{m}}}{L_{\mathrm{k}}} \ . \tag{A6}$$

Thus, we can also safely neglect the gravito-magnetic inductance $L_{\mathrm{m,G}}$ in our consideration of L_{G}.

APPENDIX B: THE KINETIC INDUCTANCE LENGTH SCALE IN A COLLISIONLESS PLASMA MODEL

In this appendix we ignore the quantum mechanical properties of superconducting films and consider the simpler, classical problem of the kinetic inductance (per square) of a thin metallic film. We begin with a physically intuitive derivation of the kinetic inductance length scale l_{k} due to D. Scalapino (whom we thank for pointing out this derivation to us). The current density for a thin metallic film is given by

$$j = n_{\mathrm{e}} e v = \frac{I}{A} = \frac{I}{wd} \ , \tag{B1}$$

where e is the electron charge, v is the average velocity of the electrons, n_{e} is the number density, A is the cross-sectional area of the film through which the current flows, w is film's width, and d is its thickness. The velocity of the electrons within the film can then be expressed as

$$v = \frac{I}{w}\frac{1}{n_{\mathrm{e}} e d} = \frac{I_{\mathrm{w}}}{n_{\mathrm{e}} e d} \ , \tag{B2}$$

where I_w is the current per width. Now, by conservation of energy it must be the case that

$$\frac{L_{\mathrm{k}} I_{\mathrm{w}}^2}{2} = \frac{m_{\mathrm{e}} v^2}{2} n_{\mathrm{e}} d \ . \tag{B3}$$

The left-hand side of (B3) gives the energy per square meter carried by the film's electrons in terms of the film's kinetic inductance per square and the square of the current per width, whereas the right-hand side gives the same quantity in terms of the kinetic energy per electron multiplied by the number of electrons per square meter of the film. Substituting (B2) into (B3) and recalling the expression for the plasma skin depth given in (34), one finds that

$$L_{\mathrm{k}} = \frac{m_{\mathrm{e}}}{n_{\mathrm{e}} e^2 d} = \mu_0 \frac{\delta_{\mathrm{p}}^2}{d} \ , \tag{B4}$$

which implies that the kinetic inductance length scale of the film is given by

$$l_{\mathrm{k}} = \frac{L_{\mathrm{k}}}{\mu_0} = d \left(\frac{\delta_{\mathrm{p}}}{d} \right)^2 \ . \tag{B5}$$

Now let us derive the kinetic inductance length scale of a thin superconducting film by treating the film as though it were a neutral, collisionless plasma consisting of Cooper-paired electrons moving dissipationlessly through a background of a positive ionic lattice. We assume that the film is at absolute zero temperature and that the mass of each nucleus in the lattice is so heavy that, to a good first approximation, the motion of the lattice in response to an incident EM wave can be neglected when compared to the motion of the electrons. If one then analyzes the film's response

to the incident EM wave using the concepts of polarization and susceptibility, it is possible to show for all non-zero frequencies that

$$\sigma_1 = 0 \quad \text{and} \quad \sigma_2 = \varepsilon_0 \frac{\omega_{\mathrm{p}}^2}{\omega} \; . \tag{B6}$$

Recalling the basic relationship between the kinetic inductance L_{k} and σ_2 given in (23), as well as the fact that $\mu_0 = 1/\varepsilon_0 c^2$, and that $\delta_{\mathrm{p}} = c/\omega_{\mathrm{p}}$ when $\omega \ll \omega_{\mathrm{p}}$, we see that according to this model the kinetic inductance of the superconducting film (in the limit of $\omega \ll \omega_{\mathrm{p}}$) $L_{\mathrm{k,p}}$ is given by

$$L_{\mathrm{k,p}} = \frac{1}{\varepsilon_0 \omega_{\mathrm{p}}^2 d} = \mu_0 \frac{\delta_{\mathrm{p}}^2}{d} \; , \tag{B7}$$

which implies that the plasma version of the kinetic inductance length scale $l_{\mathrm{k,p}}$ for a superconducting film at absolute zero is

$$l_{\mathrm{k,p}} = \frac{L_{\mathrm{k,p}}}{\mu_0} = d \left(\frac{\delta_{\mathrm{p}}}{d} \right)^2 \tag{B8}$$

in agreement with (B5). The discrepancy between these expressions and the one obtained in (37) in Section 4 on the basis of the more sophisticated BCS model,

$$l_{\mathrm{k}} = \xi_0 \left(\frac{\delta_{\mathrm{p}}}{d} \right)^2 \; , \tag{B9}$$

arises from the fact that the classical approaches taken here know nothing of the additional length scale of the BCS theory, namely, the coherence length ξ_0. This quantum mechanical length scale is related to the BCS energy gap Δ through (30) and cannot enter into derivations based solely on classical concepts; hence the appearance of the prefactor d instead of ξ_0 in (B5) and (B8).

APPENDIX C: IMPEDANCE AND SCATTERING CROSS-SECTION

The relevance of the concept of impedance to the question of scattering cross-section can be clarified by considering the case of an EM plane wave scattered by a Lorentz oscillator, which plays a role analogous to the resonant bar in Weinberg's considerations of GR-wave scattering [21]. The Poynting vector $\mathbf{S}$ of the incident EM wave is related to the impedance of free space Z_0 as follows:

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} = \frac{1}{Z_0} E^2 \hat{\mathbf{k}} \; , \tag{C1}$$

where the wave's electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$ are related to one another by $|\mathbf{E}| = Z_0 \, |\mathbf{H}|$, where $Z_0 = \sqrt{\mu_0/\varepsilon_0} = 377$ ohms is the characteristic impedance of free space, and where $\hat{\mathbf{k}}$ is the unit vector denoting the direction of the wave's propagation.

Multiplying the scattering cross-section σ (not to be confused with the conductivity) by the time-averaged magnitude of the Poynting vector $\langle S \rangle$, which is the average energy flux of the incident wave, we get the time-averaged power $\langle P \rangle$ scattered by the oscillator, viz.,

$$\sigma \langle S \rangle = \langle P \rangle = \sigma \left\langle E^2 \right\rangle / Z_0 \ , \tag{C2}$$

where the angular brackets denote a time average over one cycle of the oscillator. It follows that

$$\sigma = \frac{Z_0 \langle P \rangle}{\langle E^2 \rangle} \ . \tag{C3}$$

When driven on resonance, a Lorentz oscillator dissipates an amount of power given by

$$\langle P \rangle = \left\langle eE \frac{dx}{dt} \right\rangle = \frac{\left\langle E^2 \right\rangle}{\gamma m_{\mathrm{e}}/e^2} \ , \tag{C4}$$

where x denotes the oscillator's displacement, e is the charge of the electron, m_{e} is its mass, and γ is the oscillator's dissipation rate. The oscillator's EM scattering cross-section is thus related to Z_0 as follows:

$$\sigma = \frac{Z_0}{\gamma m_{\mathrm{e}}/e^2} \ . \tag{C5}$$

Maximal scattering will occur when the dissipation rate of the oscillator γ and thus $\gamma m_{\mathrm{e}}/e^2$ are minimized. In general, one can minimize the dissipation rate of an oscillator by minimizing its ohmic or dissipative resistance, which is a form of impedance. Hence Weinberg suggested using dissipationless superfluids instead of aluminum for the resonant bar, and we suggest here using zero-resistance superconductors instead of superfluids. In particular, Weinberg's analysis showed that if the damping of the oscillator is sufficiently dissipationless, such that radiation damping by GR radiation becomes dominant, the cross-section of the oscillator on resonance is on the order of a square wavelength, and is independent of Newton's constant G. However, the bandwidth of the resonance is extremely narrow, and is directly proportional to G.

In this regard, an important difference between neutral superfluids and superconductors is the fact that the electrical charge of the Cooper pairs enters into the interaction of the superconductor with the incoming GR wave. This leads to an enormous enhancement of the oscillator strength of Weinberg's scattering cross-section extended to the case of a superconductor in its response to the GR wave, relative to that of a neutral superfluid or of normal matter like that of a Weber bar.

As we have seen earlier, the non-localizability of the negatively charged Cooper pairs, which follows from the Uncertainty Principle and is protected by the BCS energy gap, causes them to undergo *non-geodesic* motion in contrast to the decoherence-induced *geodesic* motion of the positively charged ions in the lattice, which follows from the Equivalence Principle. The resulting

charge separation leads to a virtual plasma excitation inside the superconductor. The enormous enhancement of the conductivity that follows from this, i.e., the H-C effect, can also be seen from the infinite-frequency sum rule that follows from the Kramers-Kronig relations, which are based on causality and the linearity of the response of the superconductor to either an EM or a GR wave [2, p. 88, first equation].

In the electromagnetic sector, the Kramers-Kronig relations for the real part of the charge conductivity $\sigma_1(\omega)$ and the imaginary part $\sigma_2(\omega)$ (not to be confused with the above scattering cross-section σ) are given by [45, p. 279]

$$\sigma_1(\omega) = \frac{2}{\pi} \int_0^{\infty} \frac{\omega' \sigma_2(\omega')\, d\omega'}{\omega'^2 - \omega^2} \tag{C6a}$$

$$\sigma_2(\omega) = -\frac{2\omega}{\pi} \int_0^{\infty} \frac{\sigma_1(\omega')\, d\omega'}{\omega'^2 - \omega^2} \ . \tag{C6b}$$

From (C6b) and the fact that electrons become free particles at infinitely high frequencies, one can derive the infinite-frequency sum rule given by Kubo [45, 46]

$$\int_0^{\infty} \sigma_1(\omega) d\omega = \frac{\pi}{2} \varepsilon_0 \omega_{\mathrm{p}}^2 \ , \text{ where } \omega_{\mathrm{p}}^2 = \frac{n_{\mathrm{e}} e^2}{\varepsilon_0 m_{\mathrm{e}}} \ . \tag{C7}$$

In the GR sector, making the replacement in (C7),

$$\frac{e^2}{4\pi\varepsilon_0} \rightarrow Gm^2 \ , \tag{C8}$$

where m is regarded as the mass of the neutral atom that transports the mass current within the superfluid, is relevant to the interaction between a neutral superfluid and an incident GR wave. This leads to the following infinite-frequency sum rule:

$$\int_0^{\infty} \sigma_{1,\mathrm{G}}(\omega) d\omega = 2\pi^2 n \varepsilon_0 G m \ . \tag{C9}$$

Numerically, this result is extremely small relative to the result given in (C7), which implies a much narrower scattering cross-section bandwidth in the GR sector.

In the case of a superconductor, the replacement given by (C8) is unphysical, due to the charged nature of its mass carriers, i.e., Cooper pairs. Here, Kramers-Kronig relations similar to those given in (C6) lead to a result identical to the one given in (C7). Thus, using superconductors in GR-wave detectors will lead to bandwidths of scattering cross-sections that are orders of magnitude broader than those of neutral superfluids.

One important implication of this argument concerns the GR scattering cross-section of a superconducting sphere. If the sphere's circumference is on the order of a wavelength of an incident GR

wave, the wave will undergo the first resonance of Mie scattering. In the case of specular reflection from the surface of a superconducting sphere, this corresponds to a broadband, geometric-sized scattering cross-section, i.e., a scattering cross-section on the order of a square wavelength over a wide bandwidth. This implies that two charged, levitated superconducting spheres in static mechanical equilibrium, such that their electrostatic repulsion balances their gravitational attraction, should become an efficient transducer for converting EM waves into GR waves and vice versa [43]. As suggested in Section 10, two such transducers could be used to perform a Hertz-like experiment for GR microwaves.

ACKNOWLEDGEMENTS

We thank Jim Bardeen, Dirk Bouwmeester, Amir Caldeira, Sang Woo Chi, Victoria Chiu, Spencer De Santo, Ivan Deutsch, Uwe Fischer, Theodore Geballe, Vesselin Gueorguiev, Natalie Hall, Jim Hartle, Gary Horowitz, Boaz Ilan, Joseph Imry, Derrick Kiley, Hagen Kleinert, Don Marolf, Luis Martinez, Kevin Mitchell, Giovanni Modanese, Sir Roger Penrose, Clive Rowe, Doug Scalapino, Nils Schopohl, Achilles Speliotopoulos, Gary Stephenson, and Ken Tatebe for their many helpful comments and criticisms. R.Y.C. thanks Peter Keefe, Theo Nieuwenhuizen, and Vaclav Spicka for the invitation to speak at the *Frontiers of Quantum and Mesoscopic Thermodynamics '08* conference in Prague on the subject of this paper. This work was supported in part by a STARS Planning Grant and a STARS Research Grant from the Center for Theology and the Natural Sciences.

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[7] By "Equivalence Principle (EP)," we shall mean throughout this paper the *Weak* Equivalence Principle

(WEP), i.e., that all bodies fall with the same acceleration, when they are locally subjected to the same gravitational force (but not to any non-gravitational forces), and that their common acceleration is independent of mass, composition, thermodynamic state, quantum state, internal structure, charge, etc. This form of the EP is also known as the "Universality of Free Fall" [6]. The WEP is to be distinguished from the stronger form of the EP known as the "Einstein Equivalence Principle (EEP)," which adds Poincaré invariance to the above statement of the WEP. See C.M. Will, Living Rev. Relativity 9 (2006) 3.

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[9] A free particle initially at rest will remain at rest during and after the passage of a weak (linearized) GR wave. A particle can absorb energy from a GR wave "only if (i) it is interacting with other particles, or (ii) it has nonzero energy initially, or (iii) the wave is strong so that nonlinear effects are important." S.A. Teukolsky, Phys. Rev. D 26 (1982) 745.

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[16] It should be noted that a superconductor's normal conduction electrons are just as localizable within the material as the ions, since no energy gap exists in their case. The behavior of normal conduction electrons is said by condensed-matter physicists to be "delocalized" with respect to a given ion within a few lattice constants, but these electrons are not "nonlocalizable" in the sense of the Cooper pairs discussed here.

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[21] S. Weinberg, Gravitation and Cosmology, Wiley, New York, 1972.

[22] Tinkham [2] uses a different convention, namely, $\sigma_s(\omega) = \sigma_{1\mathrm{s}}(\omega) - i\sigma_{2\mathrm{s}}(\omega)$, which arises from his choice for time dependence to be $\exp(+i\omega t)$, whereas we employ a time dependence of $\exp(-i\omega t)$, so that here $\sigma_{\mathrm{s}}(\omega) = \sigma_{1\mathrm{s}}(\omega) + i\sigma_{2\mathrm{s}}(\omega)$.

[23] The electron's *negative* charge means that the charge current density $\mathbf{j}_{\mathrm{e}}$ flows in a direction *opposite* to the velocity vector $\mathbf{v}$ of the electrons, whereas the electron's *positive* mass means that the mass-current density $\mathbf{j}_{\mathrm{G}}$ flows in the *same* direction as $\mathbf{v}$. Hence the mass-current density $\mathbf{j}_{\mathrm{G}}$ flows in a direction *opposite* to the charge current density $\mathbf{j}_{\mathrm{e}}$. But because of the negative sign in front of the mass current density source term on the right-hand side of the gravitational Ampere-like law in the Maxwell-like equations (40d), $\nabla \times \mathbf{B}_{\mathrm{G}} = -\mu_{\mathrm{G}}\mathbf{j}_{\mathrm{G}}$, there is a minus sign on the right-hand side of (8) in the GR case, which cancels the former minus sign, so that on the right-hand side of (12) and in all subsequent equations for the GR case, the signs remain unchanged. The minus sign in front of the mass-current-density source term on the right-hand side of the Ampere-like law (40d) in the Maxwell-like equations stems from the fact that, when one takes the divergence of both sides of this equation, one must obtain zero, since the divergence of a curl is zero. This will indeed be the result, provided that both the continuity equation for the mass current density and the Gauss-like law (40a) in the Maxwell-like equations are simultaneously satisfied on the right-hand side of the resulting equation. The minus sign in front of the mass-density source term in the Gauss-like law is necessitated by the experimental fact that the gravitational force between all masses is attractive, which implies that the minus sign in front of the mass-current-density source term in the Ampere-like law (40d) is also necessitated by the same experimental fact. These two minus signs also emerge naturally from the linearization of Einstein's field equations.

[24] This formula can also be understood in terms of a simple transmission line model, where the line has a characteristic impedance Z and a purely reactive shunt element iX, where the element iX could represent the effect of the kinetic inductance of the film. We wish to calculate the power reflected from this element when an incident wave propagates down the transmission line. The equivalent circuit consists of the same transmission line terminated by means of a parallel combination of the reactive element and a resistor having a resistance Z, which is a real number. The reflectivity of this configuration is given by

$$\mathcal{R} = |\rho|^2 = \rho^* \rho = \left| \frac{Z - Z_{\mathrm{eff}}}{Z + Z_{\mathrm{eff}}} \right|^2$$

where

$$\rho = \frac{Z - Z_{\mathrm{eff}}}{Z + Z_{\mathrm{eff}}}$$

and

$$Z_{\text{eff}} = \frac{iX \cdot Z}{iX + Z} \; .$$

From this it follows that

$$\begin{aligned} \rho &= \left(Z - \frac{iX \cdot Z}{iX + Z} \right) \left(Z + \frac{iX \cdot Z}{iX + Z} \right)^{-1} \\ &= \frac{1}{1 + 2iX/Z} \end{aligned}$$

and that

$$\begin{aligned} \mathcal{R} &= \left(\frac{1}{1 - 2iX/Z} \right) \left(\frac{1}{1 + 2iX/Z} \right) \\ &= \left\{ 1 + \left(2\frac{X}{Z} \right)^2 \right\}^{-1} \; . \end{aligned}$$

With the identifications of $X = X_{\text{L}}$ and $Z = Z_0$, we see that the reflectivity of the reactive element has the same form as the EM reflectivity of the superconducting film given by (24).

[25] The Drude model involves an equation of motion for the expectation value of the electron velocity, namely

$$m_{\text{e}} \frac{d}{dt} \langle \mathbf{v}_{\text{e}} \rangle = \frac{-m_{\text{e}}}{\tau} \langle \mathbf{v}_{\text{e}} \rangle - e\mathbf{E} \; ,$$

where one adopts a semi-classical approach in which the electrons are treated quantum-mechanically and the external fields classically. This Langevin-like equation follows from the Fokker-Planck equation, which models diffusion in phase-space. The mean free time τ in the Drude model is assumed to be d/v_{F} where d is the thickness of the film and v_{F} is the Fermi velocity. The steady-state solution of this model is given by

$$\langle \mathbf{v}_{\text{e}}(t \to \infty) \rangle = \frac{\tau e}{m_{\text{e}}} \mathbf{E} \; .$$

If we now use the usual expression for the current density

$$\langle \mathbf{j}_{\text{e}} \rangle = n_{\text{e}} e \langle \mathbf{v}_{\text{e}} \rangle$$

where n_{e} is the number density of the electrons, we see that

$$\langle \mathbf{j}_{\text{e}} \rangle = \frac{n_{\text{e}} e^2 d}{m_{\text{e}} v_{\text{F}}} \mathbf{E} \; .$$

Equating σ_{n} with the prefactor on the right-hand side, we recover Ohm's law and arrive at the expression given in (27) for the normal conductivity.

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[28] R.M. Wald, General Relativity, University of Chicago Press, Chicago, 1984. The Maxwell-like equations are to be supplemented by the Lorentz-like force law

$$\mathbf{F}_{\text{G}} = m_{\text{test}} \left(\mathbf{E}_{\text{G}} + 4\mathbf{v} \times \mathbf{B}_{\text{G}} \right) \; ,$$

where m_{test} is the mass of a local test particle moving at a non-relativistic velocity $\mathbf{v}$. The Maxwell-like approach derived by Wald (as well as the approach used by DeWitt in [29]) is fully equivalent to

the standard transverse-traceless approach used by the GR community for GR wave physics, since a simple coordinate transformation based on a local Galilean transformation exists between the Wald gauge and the transverse-traceless (TT) gauge. A transformation of our Wald-based calculations into the standard TT gauge will not change the value of the ratio given by (1). The necessarily quadrupolar nature of the mass and the mass-current sources in Wald's equations, which originates physically from the fact that there is only one sign of mass and which follows from the EP, excludes the usual, uniform, crossed electric- and magnetic-field EM plane-wave solutions. Thus the lowest-order allowed plane-wave solutions of Wald's equations in the vacuum, which are homologous to the TEM_{11} solutions for EM plane waves propagating in the vacuum, will induce a *quadrupolar* pattern of mass supercurrents in a superconducting film at normal incidence.

[29] B.S. DeWitt, Phys. Rev. Lett. 16 (1966) 1092.

[30] Our focus on mass currents should put to rest any concerns that we are claiming that static longitudinal gravito-electric fields can be shielded by matter, i.e., that anti-gravity is possible. This would require the existence of negative mass particles in the vacuum, which has never been observed. Our claim is rather that time-varying transverse gravito-electric fields can be reflected by the mass currents generated within superconductors by GR waves. Mass currents of opposite signs, i.e., mass currents flowing in opposite directions, *can* exist in nature. Time-varying mass currents generate time-varying gravito-magnetic fields, which can in turn reflect time-varying, transverse gravito-electric fields.

[31] C. Kiefer, C. Weber, Ann. der Phys. (Leipzig) 14 (2005), 253. The earliest mention of the concept of the characteristic gravitational impedance of free space, which was based on a dimensional analysis that resulted in an estimate of Z_{G} based on a ratio of Newton's constant G to the speed of light c, was perhaps made by J.D. Krauss (private communication).

[32] R.Y. Chiao, W.J. Fitelson, A.D. Speliotopoulos, arXiv:gr-qc/0304026; R.Y. Chiao, W.J. Fitelson, in Proceedings of the Time & Matter Conference, Venice, Italy, 11-17 August 2002, I. Bigi and M. Faessler (Eds.), World Scientific, Singapore, 2006; arXiv:gr-qc/0303089.

[33] The relationship between the gravitational vector potential $\mathbf{h}$ used in (59a), which has units of velocity, and the usual dimensionless strain amplitude h_+ used, for example, by P.R. Saulson, Class. Quantum Grav. 14 (1997) 2435, is as follows. The GR-wave flux, which follows from (151), is

$$\langle S_{\mathrm{GR}}\rangle = \frac{1}{2Z_{\mathrm{G}}}\left\langle E_{\mathrm{G}}^2\right\rangle \ .$$

Using the fact that $\mathbf{E}_{\mathrm{G}} = -\partial\mathbf{h}/\partial t$, and assuming sinusoidally-varying incident radiation, this leads to

$$\langle S_{\mathrm{GR}}\rangle = \frac{\omega^2}{2Z_{\mathrm{G}}}\left|\mathbf{h}\right|^2 \ .$$

Comparing this with Saulson's expression for the gravitational wave flux and using the value of Z_{G} given in (44), we see that

$$\langle S_{\mathrm{GR}}\rangle = \frac{\omega^2 c^2}{8Z_{\mathrm{G}}}\left|h_+\right|^2 \ ,$$

and thus that

$$|h_+| = 2\frac{|\mathbf{h}|}{c} \ .$$

[34] In the case of a plane wave $\psi = \sqrt{n}\exp(i\mathbf{k}\cdot\mathbf{r})$, the probability current density given in (57) becomes

$$\mathbf{j} = \frac{\hbar}{2mi}(\psi^*(i\mathbf{k})\psi - \psi(-i\mathbf{k})\psi^*) = n\frac{\hbar}{m}\mathbf{k} = n\mathbf{v} \ .$$

Thus the speed v associated with the current density j is

$$v = \frac{\hbar}{m}k \ .$$

Now, the dispersion relation for de-Broglie matter waves is given by

$$\omega = \frac{\hbar k^2}{2m} \ ,$$

which leads to

$$\begin{aligned} v_{\text{group}} &\equiv \frac{d\omega}{dk} = \frac{\hbar}{m}k \\ v_{\text{phase}} &\equiv \frac{\omega}{k} = \frac{1}{2}\frac{\hbar}{m}k \ . \end{aligned}$$

Hence the velocity $\mathbf{v}$ given in (69), which is associated with the probability current density $\mathbf{j}$ given in (57), is the *group* velocity, and not the *phase* velocity, of a Cooper pair.

[35] Note that the $\mathbf{j}_{\text{G}}$ and $\mathbf{F}_{\text{tot}}$ will be nonzero as long as the Schiff-Barnhill effect [29]

$$q\mathbf{E} + m\mathbf{g} = \mathbf{0} \ ,$$

which is present in a normal metal, is absent in the superconductor. The static mechanical equilibrium represented by the equation above is never achieved inside a superconductor, since the establishment of such equilibrium requires the action of dissipative processes that are present in a normal metal but absent in a superconductor. In particular, we should expect a large breakdown of the Schiff-Barnhill effect at microwave frequencies, when plasma oscillations of the superconductor can in fact be virtually excited purely electromagnetically.

[36] Only a certain amount of current can exist in the film before it is driven normal by the resulting magnetic field. It is therefore important to take into account the critical field of the superconducting film when considering the linear response of the film to any incident radiation. In the EM sector, it can be shown using the Poynting vector that an incident wave-power flux density of 10 Watts per square meter will induce a magnetic field of 2 milli-Gauss, which is five orders of magnitude below the critical magnetic field of lead at temperatures well below T_c. Alternatively, lead will remain superconducting for incident power flux densities below tera-Watts per square meter. The critical magnetic field of a superconductor is related to the Helmholtz free energy density of the superconducting state stored in the condensate [2], which in turn is related to the BCS energy gap. This gap is related only to quantities that correspond to intrinsic properties of the superconductor, and not to any coupling to external fields. Exactly the same considerations will thus apply in the GR sector. The critical *gravito*-magnetic field of a superconductor will correspond to the same free energy density as the critical magnetic field, and a GR wave containing a power flux density of 10 Watts per square meter will produce a gravito-magnetic field that lies equally far below the critical gravito-magnetic field.

[37] One should regard (91) as a quantum-mechanical operator equation of motion for the Cooper pair's charge density operator $\rho_{\rm e}$ and mass density $\rho_{\rm G}$ operator, which are related to the quantum-mechanical density operator $\rho = |\psi\rangle\langle\psi|$ by

$$\rho_{\rm e} = q\,|\psi\rangle\langle\psi| \text{ and } \rho_{\rm G} = m\,|\psi\rangle\langle\psi| \ ,$$

respectively. Similarly, the probability current density $\mathbf{j}$, the quantum velocity field $\mathbf{v}$, and the acceleration field $\mathbf{a}$ of the superconductor are also quantum operators.

[38] This applies to non-zero frequencies only, since at DC, the mass conductivity $\sigma_{1,\rm G}$, like the charge conductivity, is infinite, i.e., where

$$\sigma_{1,\rm G} = A_{\rm G}\delta(\omega)$$

for some constant $A_{\rm G}$.

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